Constant Approximation for Capacitated k-Median with (1+ϵ)-capacity Violation

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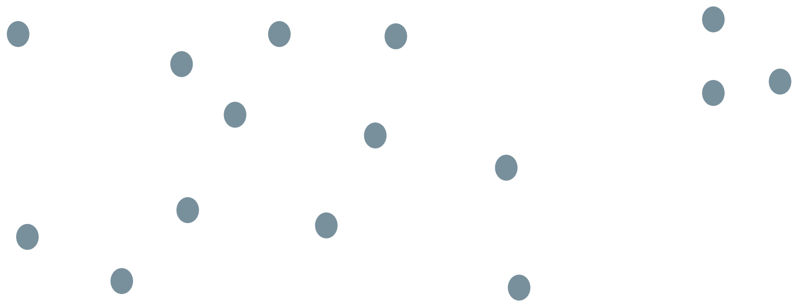
k-Median

Input:

- Set of clients ***C***,



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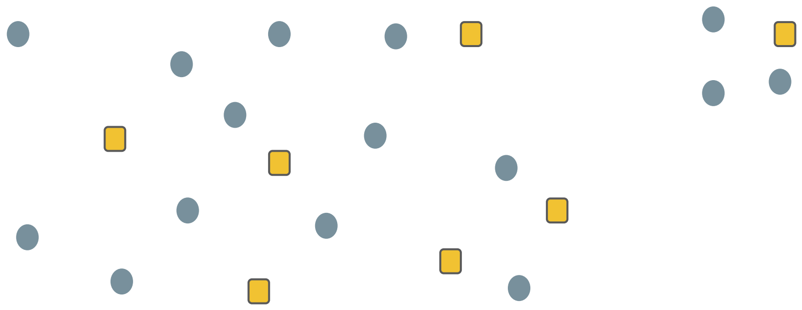

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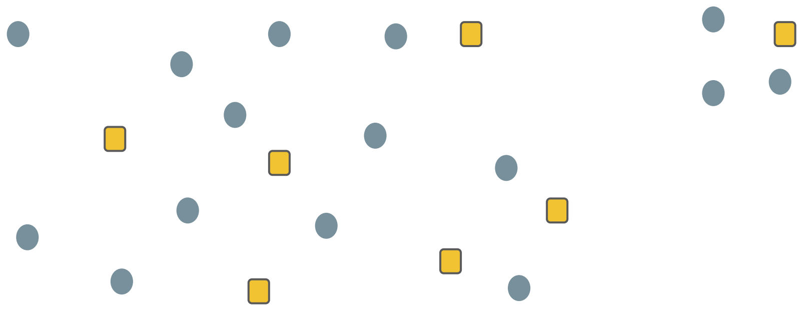

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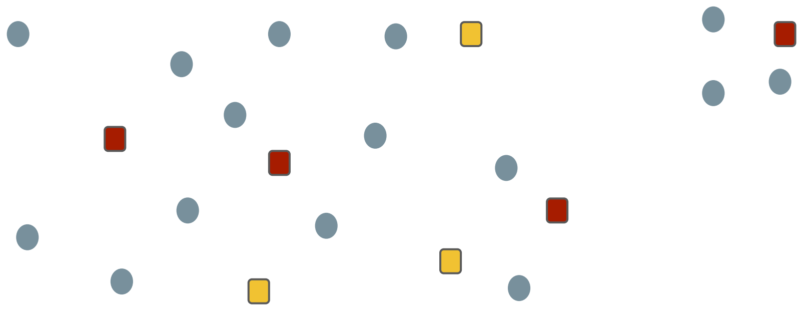

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Input:

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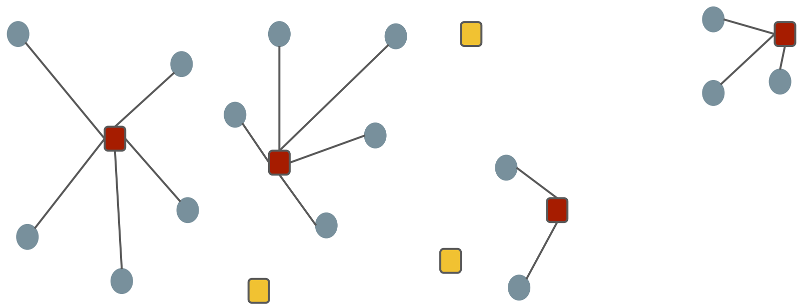

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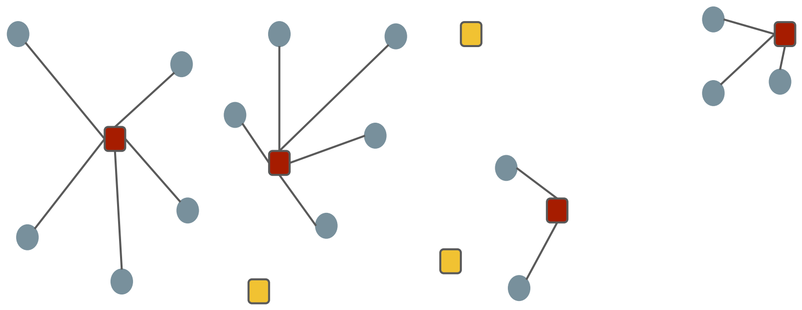

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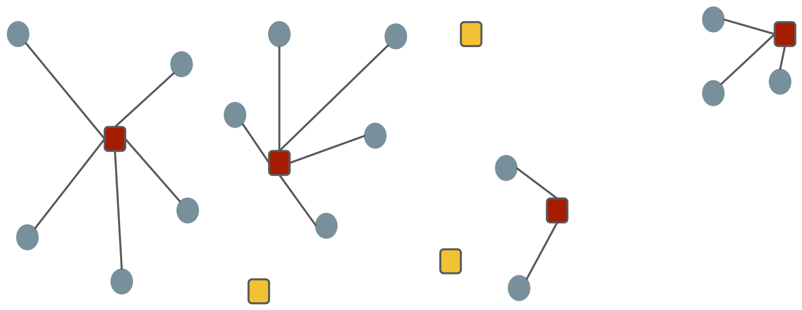
Objective: Min total connection distance

$$\sum_{j \in C} \mathbf{d}(\boldsymbol{\sigma}(j), j)$$



Capacitated *k*-Median

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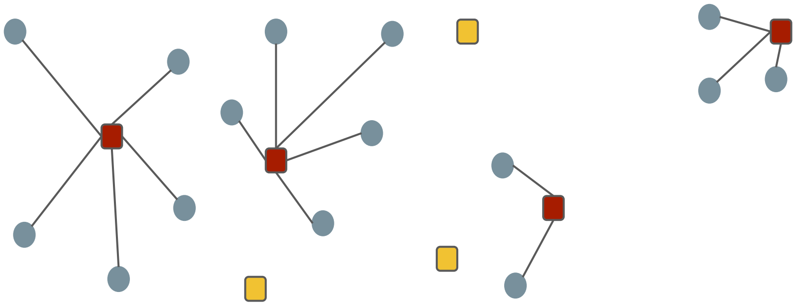

Capacitated *k*-Median

Input:
- Set of clients C,
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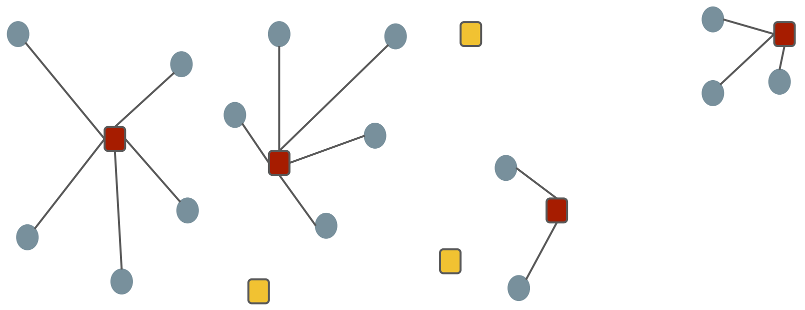

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Input:

- Set of clients ***C***,
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- Metric **d** on $\boldsymbol{F} \cup \boldsymbol{C}$,
- Integer **k**.

Output:

- Open facilities $F' \subseteq \boldsymbol{F}$
- Connect $\boldsymbol{\sigma}$: $\boldsymbol{C} \rightarrow F'$

Constraint:

- $|F'| \leq \mathbf{k}$ (cardinality cons.)
- $|\boldsymbol{\sigma}^{-1}(i)| \leq \mathbf{u}_i$ (capacity cons.)

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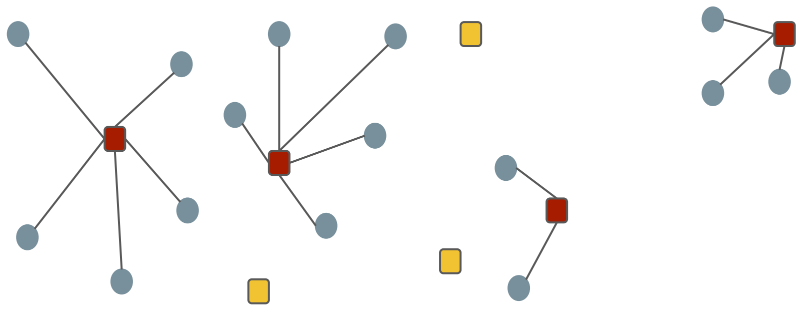

Basic Linear Program

$y_i = 1$: facility $i \in \boldsymbol{F}$ is open

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min	$\sum_{i \in \boldsymbol{F},\, j \in \boldsymbol{C}} x_{i,j}\, \mathbf{d}(i, j)$	
(cardinality constraint)	$\sum_{i \in \boldsymbol{F}} y_i \le \mathbf{k}$,	
	$\sum_{i \in \boldsymbol{F}} x_{i,j} = 1$	$\forall j \in \boldsymbol{C}$,
(capacity constraint)	$\sum_{j \in \boldsymbol{C}} x_{i,j} \le \mathbf{u_i} y_i$	$\forall i \in \boldsymbol{F}$,
	$x_{i,j} \le y_i$	$\forall j \in \boldsymbol{C}, i \in \boldsymbol{F}$
	$0 \le x_{i,j}, y_i \le 1$	$\forall j \in \boldsymbol{C}, i \in \boldsymbol{F}$

Basic LP has unbounded integrality gap!

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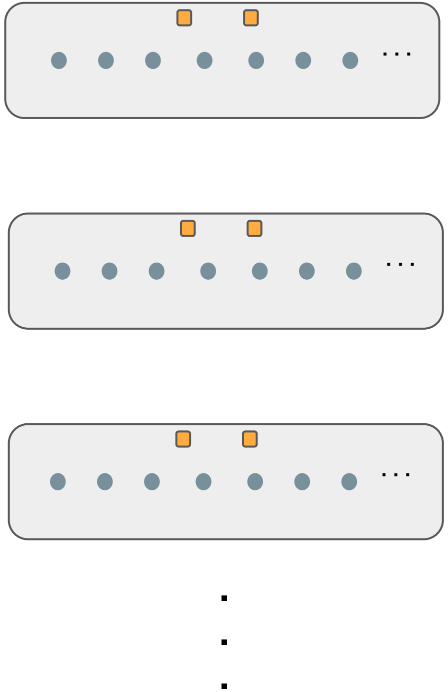
Idea: Isolated groups!



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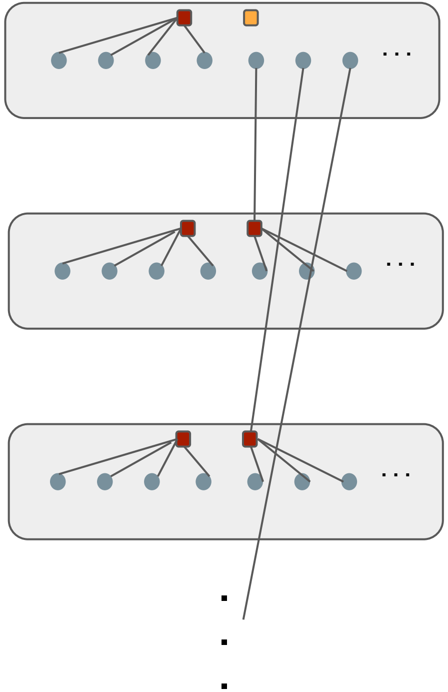
Costly



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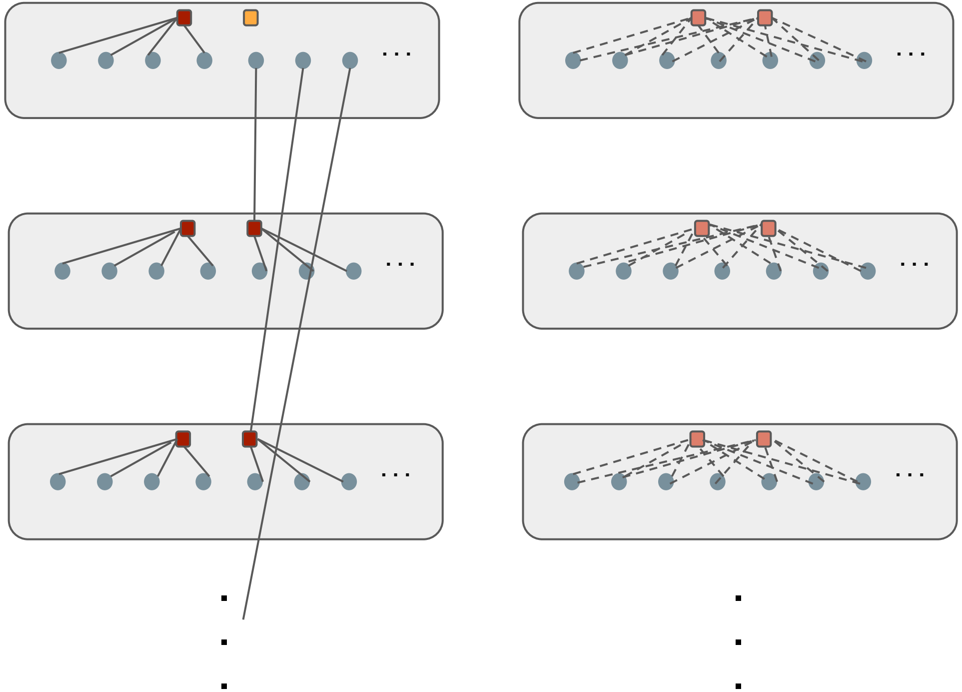

Basic LP fractional solution:
No Cost!

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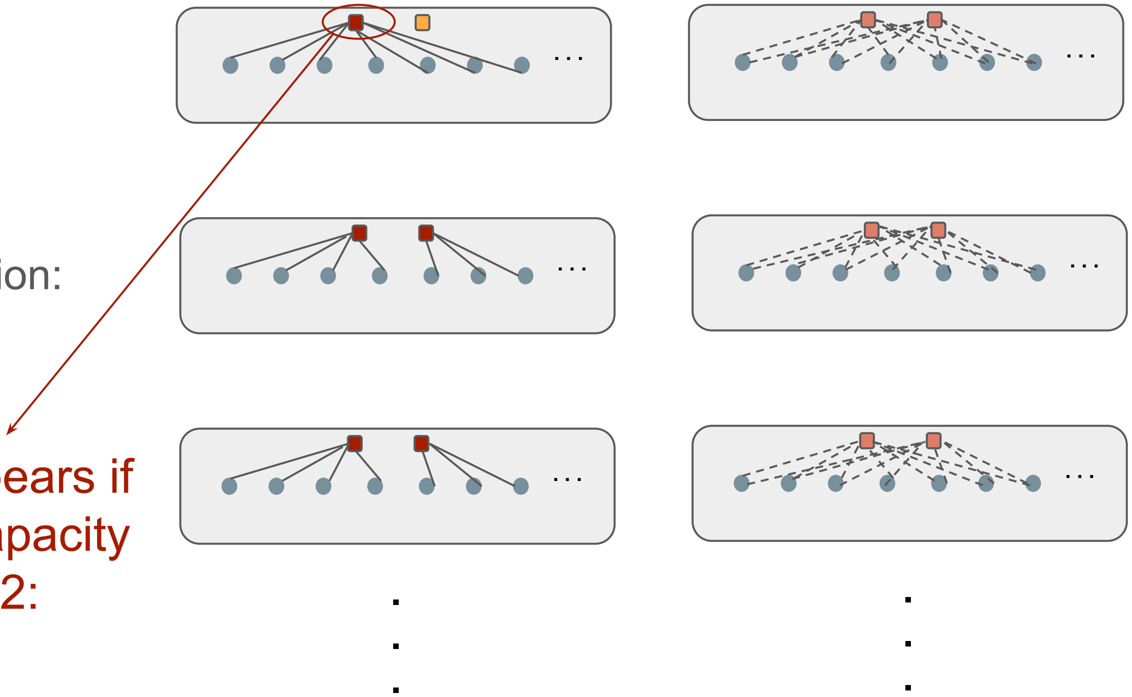

Integral solution: Costly

Gap disappears if we allow capacity violation of 2: **2u**

Basic LP fractional solution: No Cost!

Status of Capacitated ***k***-Median

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Violate capacity constraint by a factor α (connect α**u** clients)

For Basic LP, α must be ≥ 2

Status of Capacitated ***k***-Median

Pseudo approximations with cardinality (**k**) violation:

Cardinality violation factor	Approx Factor		Technique
$12+17/\epsilon$	$1+\epsilon$	[KPR'98]	Local Search
$5+\epsilon$	$O(1/\epsilon^3)$	[KPR'98]	Local Search
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$1+\epsilon$	$O(1/\epsilon^2 \log 1/\epsilon)$	[Li'15]	Configuration LP

Limit of Basic LP

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Pseudo approximations with capacity (u) violation:

Capacity violation factor	Approx Factor		Technique

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40	50	[CR'05]	+Dual fitting
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Our Result:	1+ϵ	$O(1/\epsilon^5)$		Configuration LP

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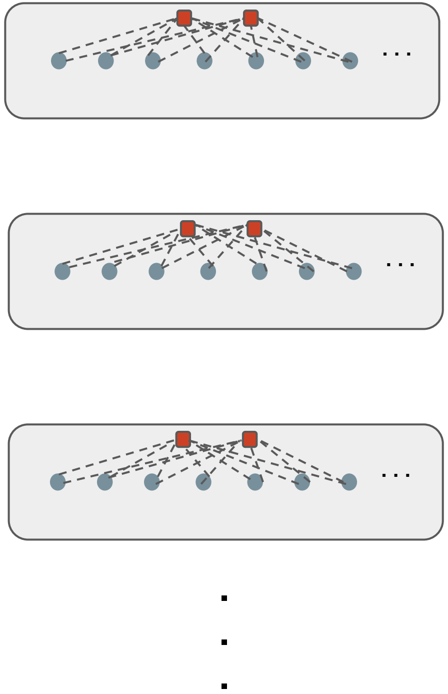
Configuration LP - intuition



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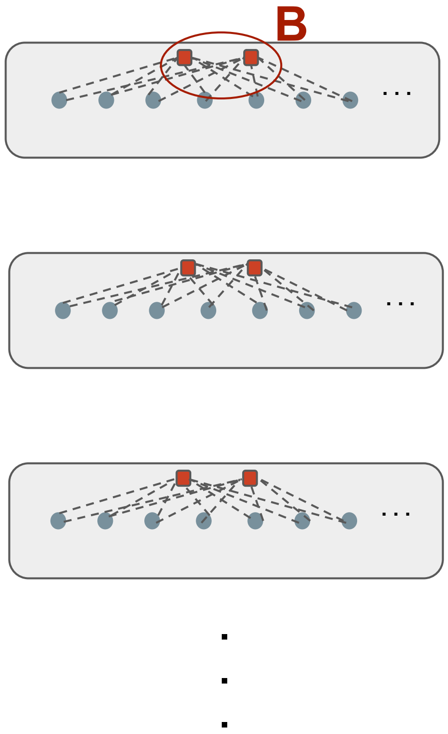

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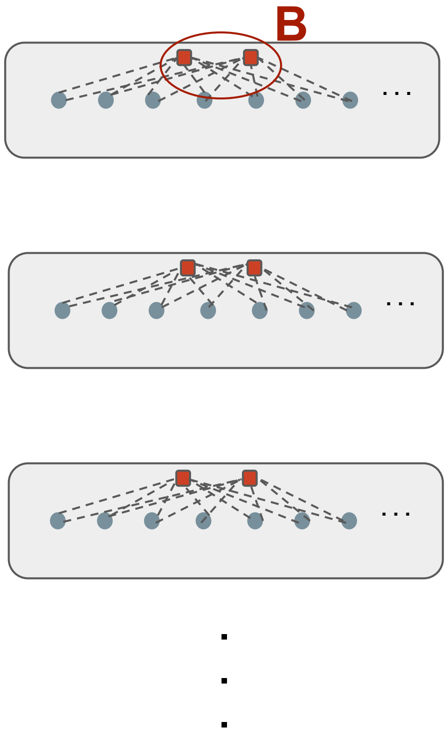

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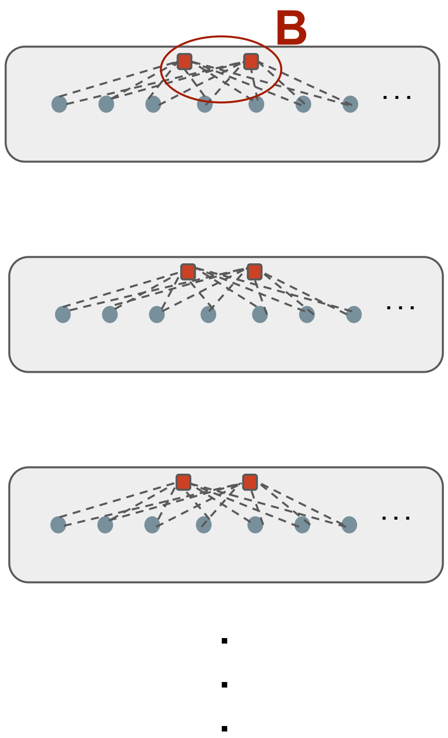

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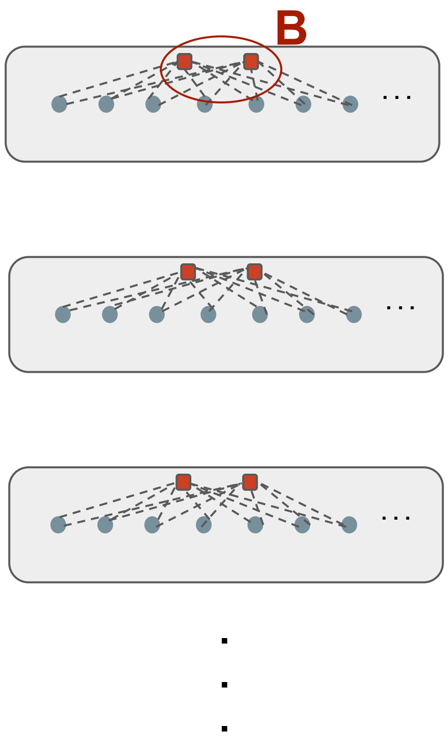

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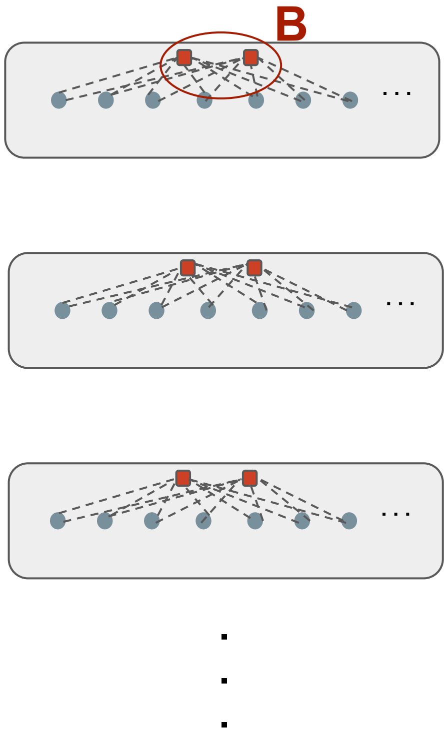

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Goal: get "integral" solutions for B if $y_{\boldsymbol{B}}$ small

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$\forall B \subseteq F$, introduce variables $z_\perp^B$ and $\{z_S^B\}$

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Our algorithm either rounds or finds a violated constraint for ellipsoid alg.!

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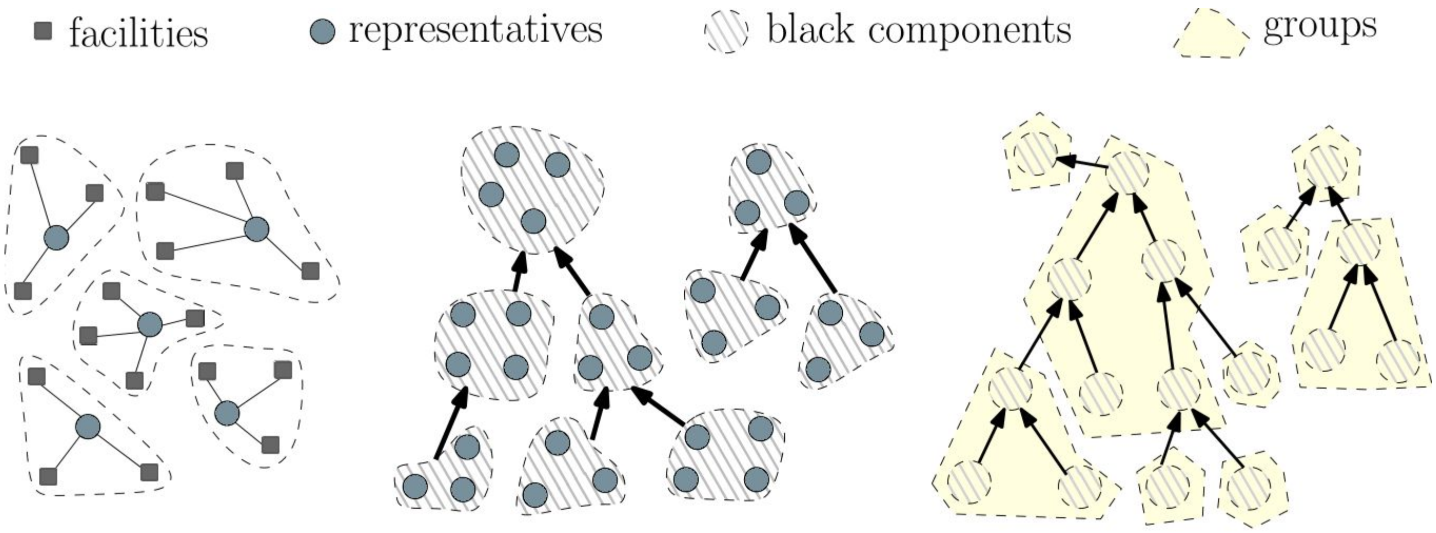

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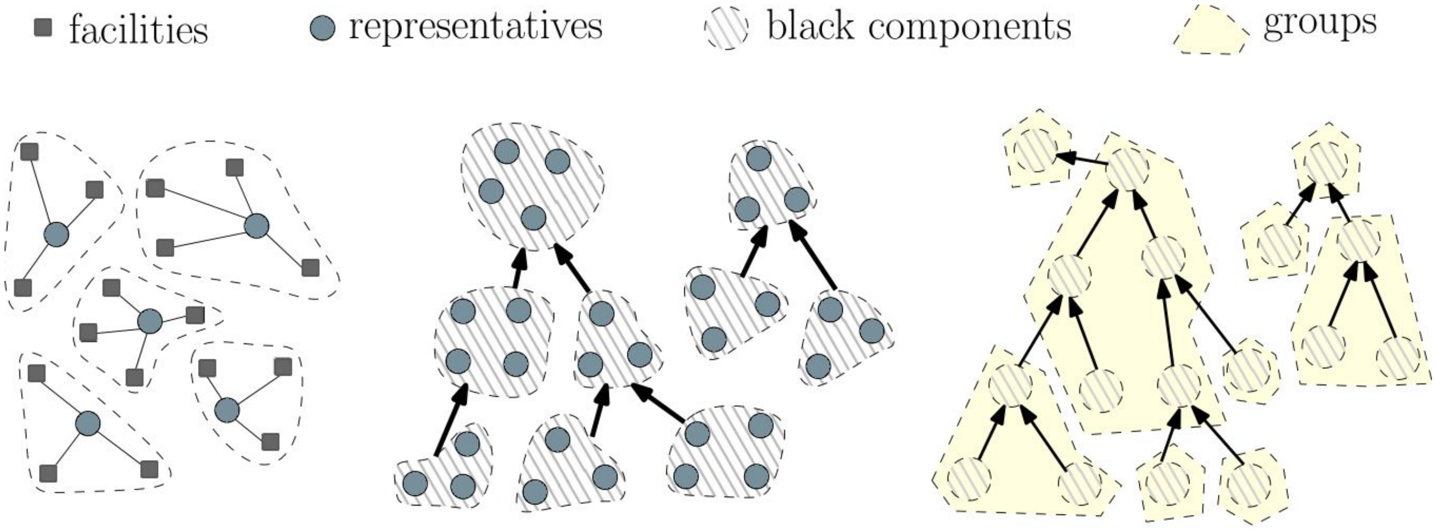

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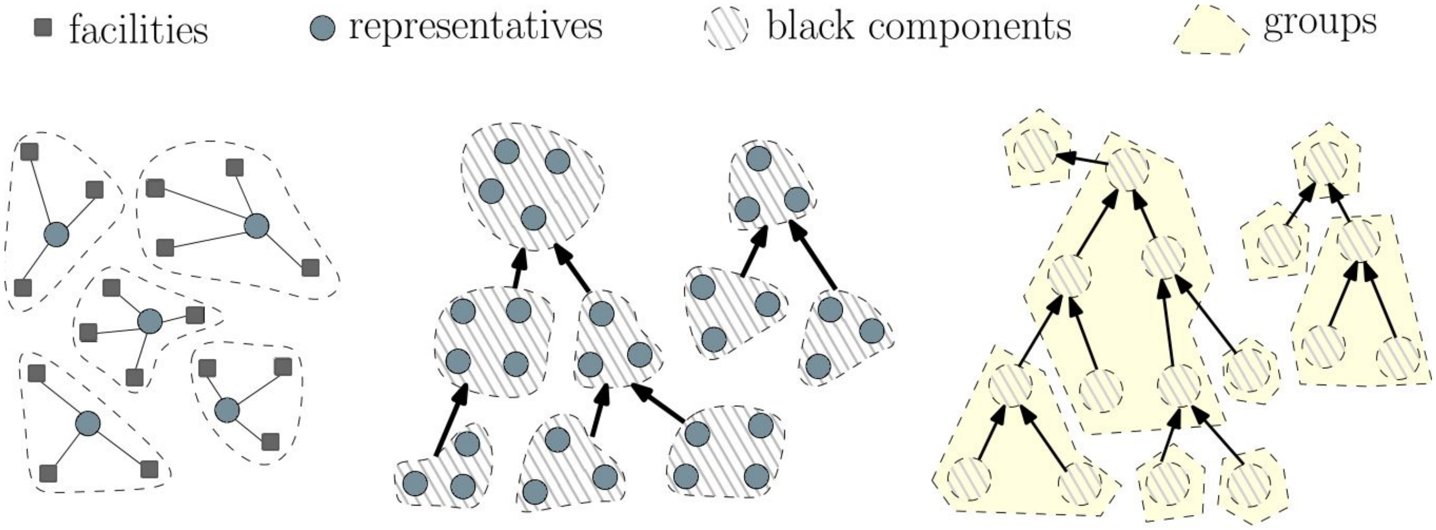

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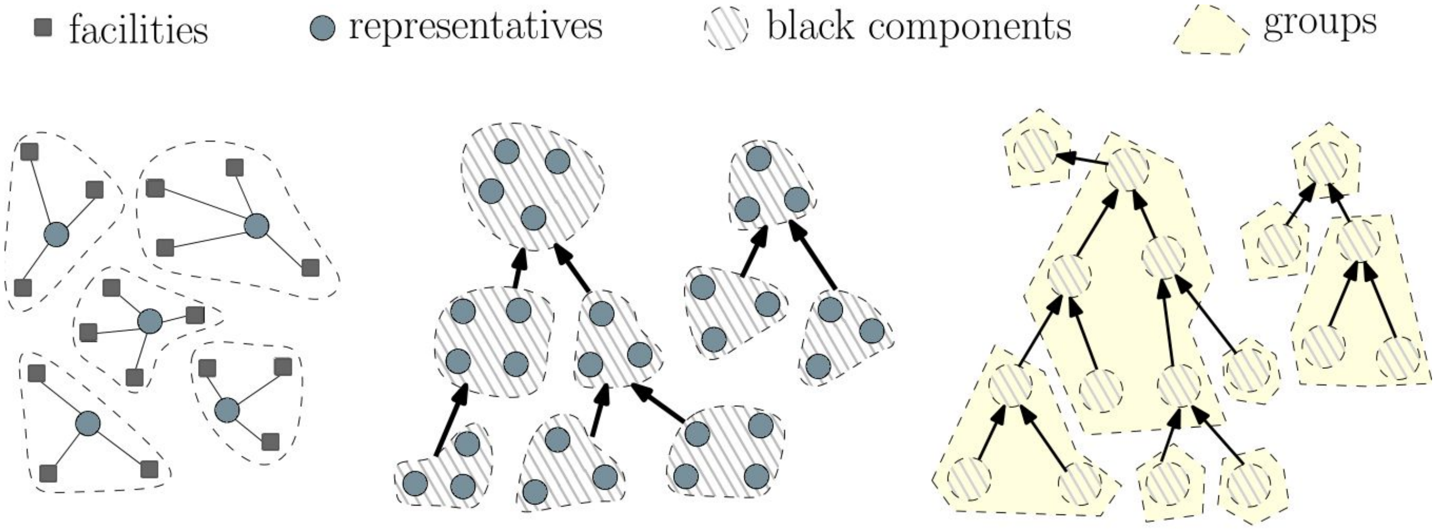

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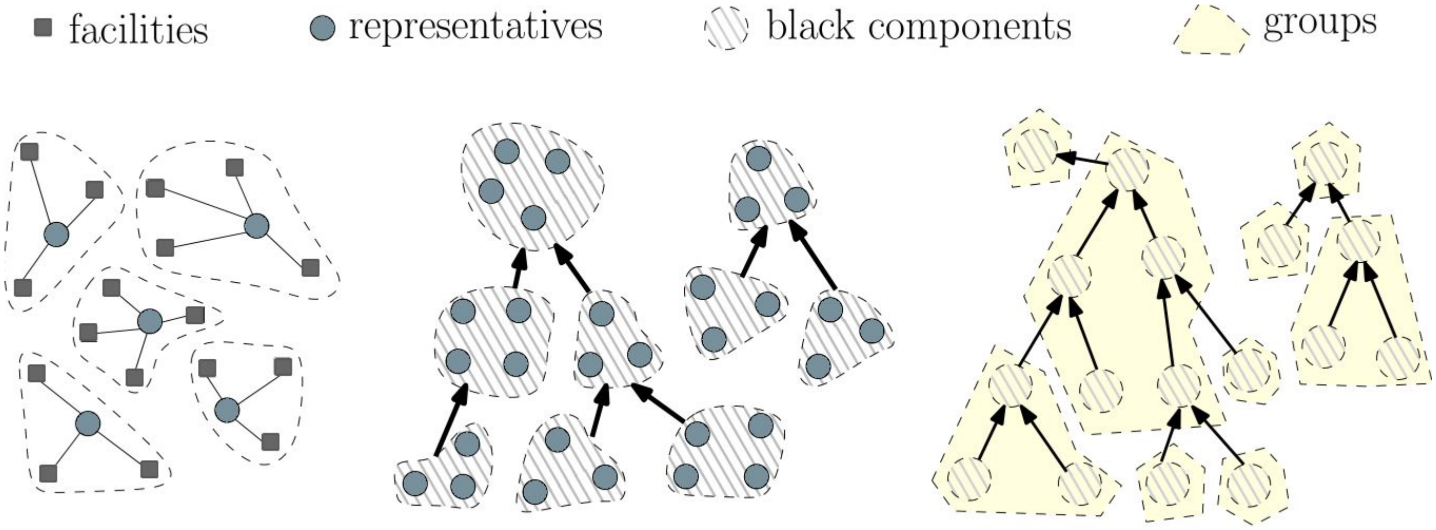

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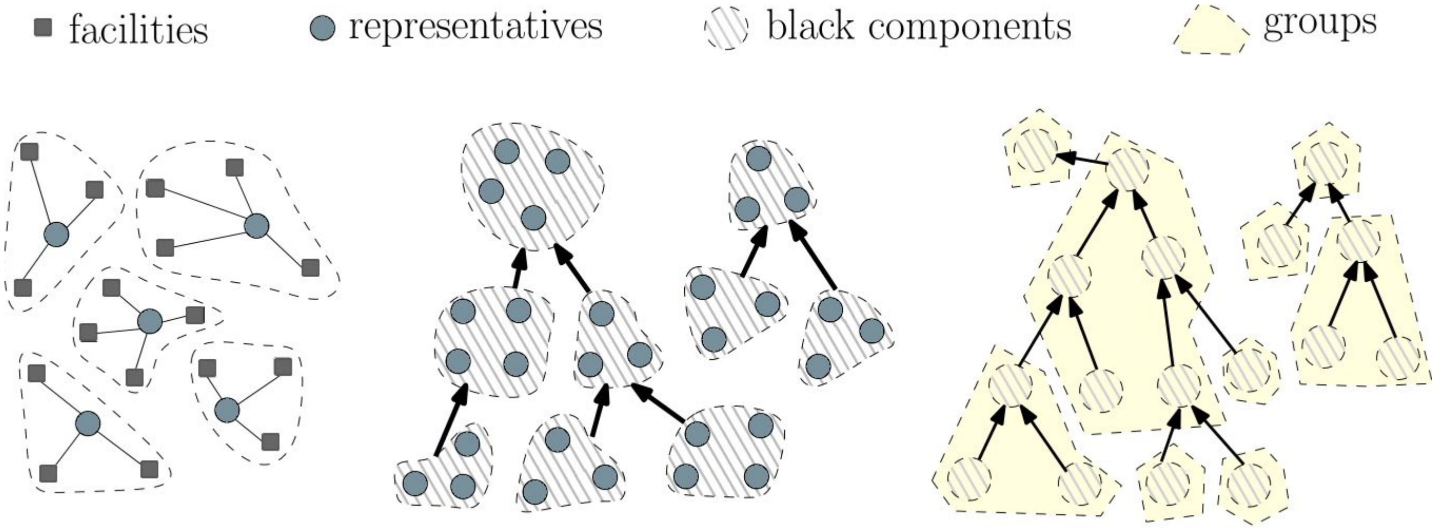

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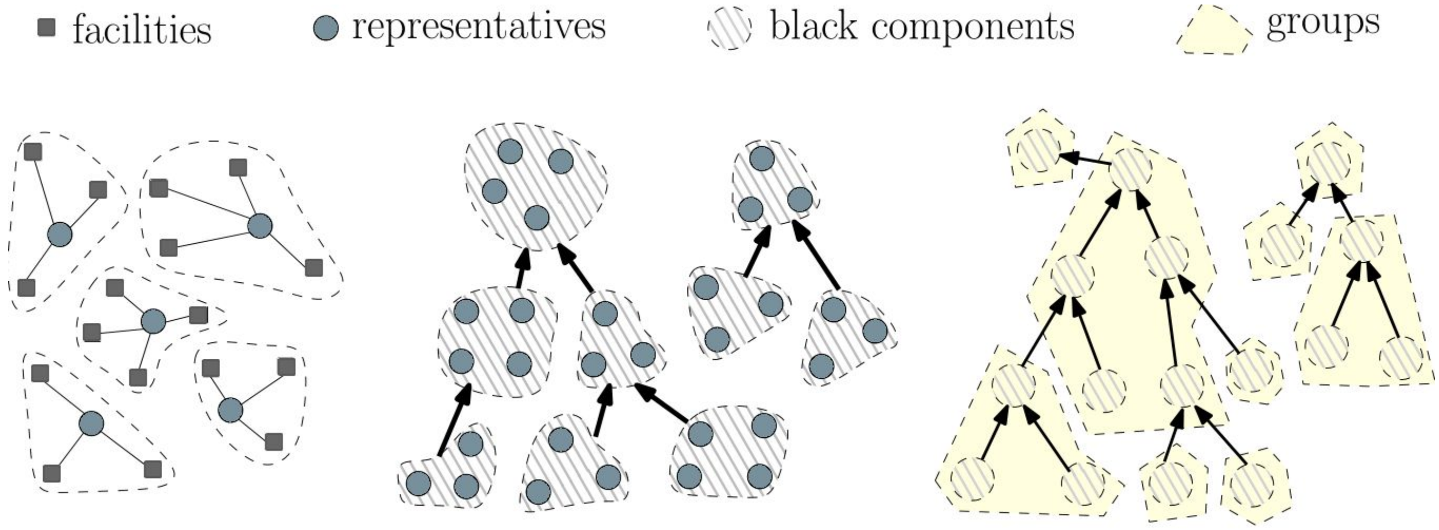

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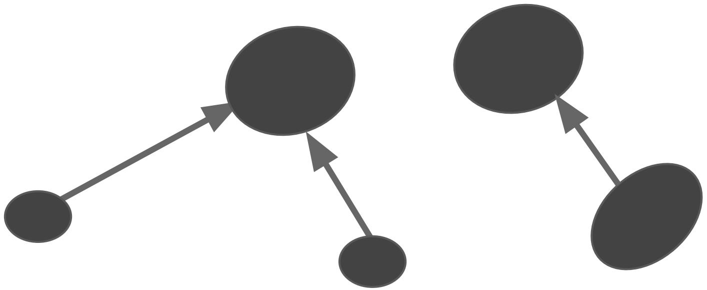
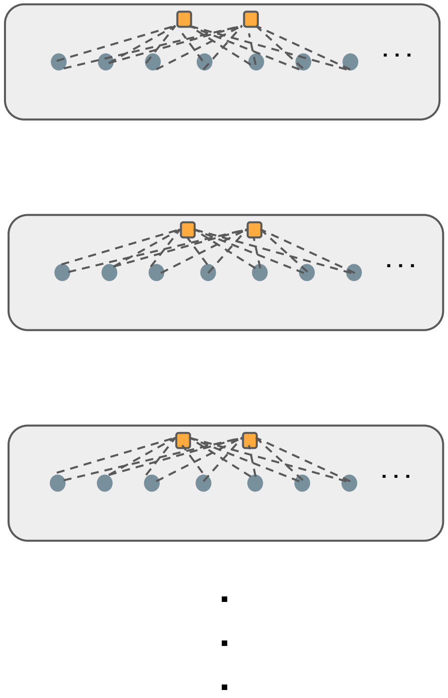
Defining Concentrated Components

Black component → Concentrated

Black component → Non-Concentrated





Defining Concentrated Components

Extreme case:

- A client is either fully connected to a black comp. J

  e.g. $x_{J,j} = 1$

  or fully connected to components other than J

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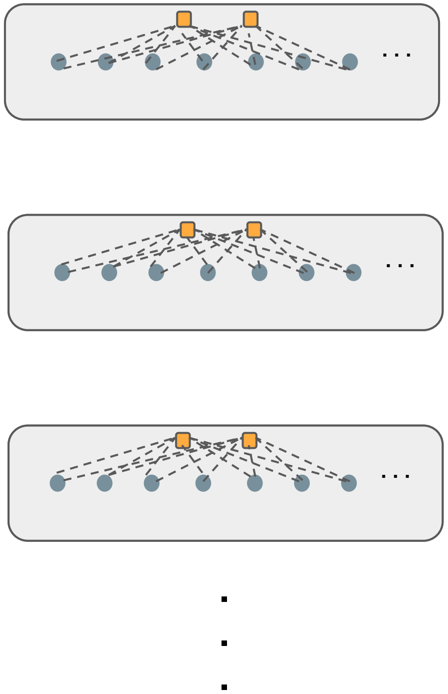

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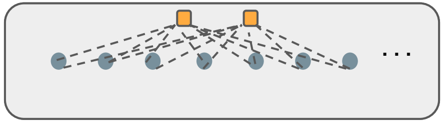

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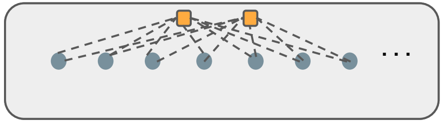
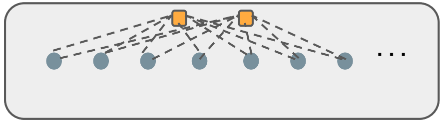


More smooth:

- Define $\pi_J := \sum_{j \in \mathcal{C}} (1-x_{J,j}) x_{J,j}$ for a black comp. J



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Defining Concentrated Components

- Define $\pi_J := \sum_{j \in \mathcal{C}} (1-x_{J,j})x_{J,j}$ for a black comp. J
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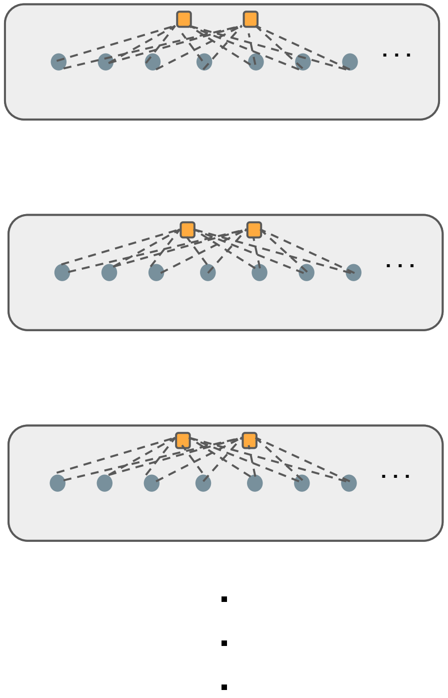

Defining Concentrated Components

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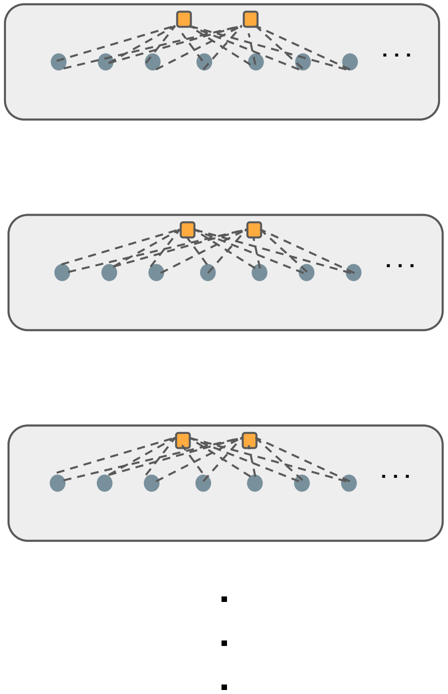

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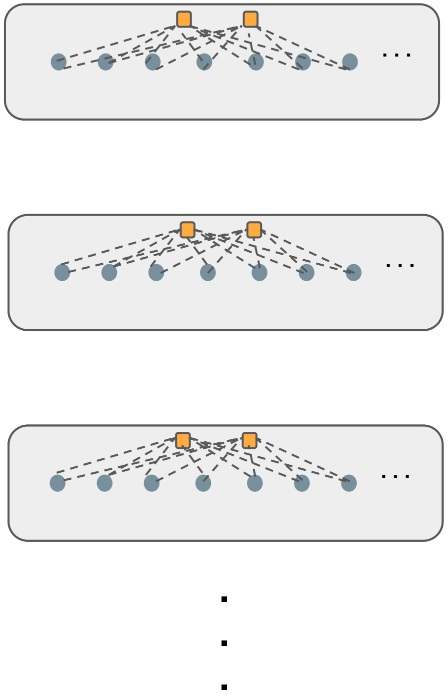

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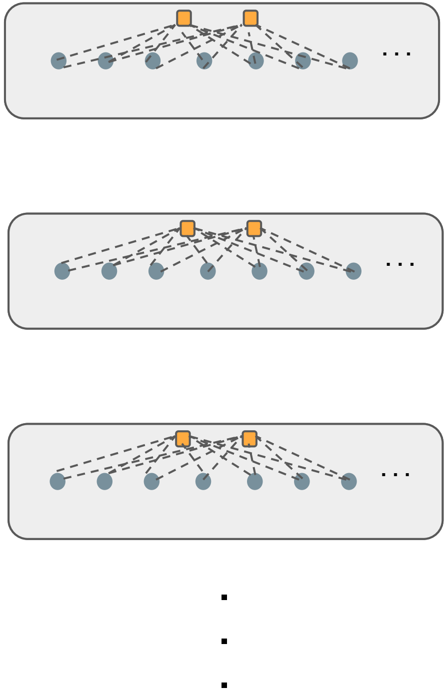

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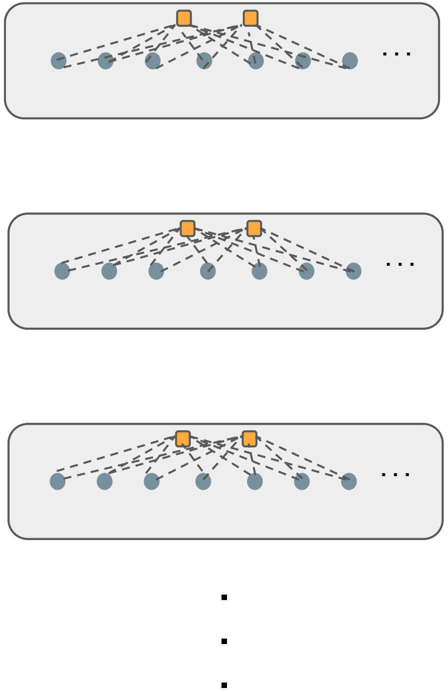
  - We can carry **all demand** out with $1/\epsilon^3$ $Cost_{LP}$



What's next?

- Configuration LP
- Rounding algorithm for (1+ϵ) capacity violation
  - 3-phase Clustering
  - Obtaining Local Solutions
    - Defining Concentrated (isolated) Components
    - Distributions over Local Solutions for Concentrated Components
  - Putting it all together

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Distributions Over Local Solutions for Concentrated Components

- Basic LP solution is NOT sufficient (gap example)
- For each concentrated component J,
  - If Configuration LP constraints are NOT satisfied for J, return a constraint not satisfied to ellipsoid algorithm
  - o/w use z_S's for each small $S \subseteq J$ get a "raw" distribution over solutions

$$z_\perp + \sum_S z_S = 1$$

Distributions Over Local Solutions for Concentrated Components

- We'll extract a distribution over "nice" integral solutions from $\{z_S\}$, $\{z_{S,i}\}$, $\{z_{S,i,j}\}$

(raw distribution: expected number of open facilities $y_{\boldsymbol{B}}$, expected amount of demand served $x_{\boldsymbol{B,C}}$)

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Idea: Use that this is a concentrated component!

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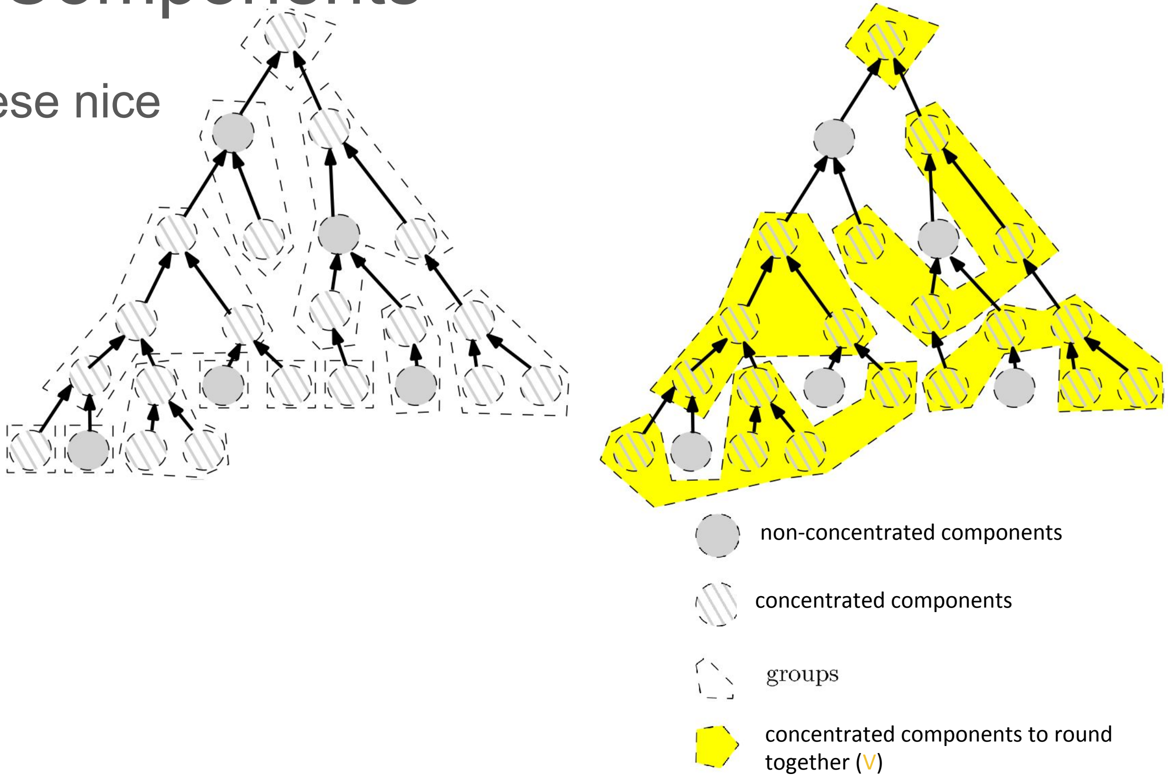
first O(ϵ) capacity blow up

Distributions Over Local Solutions for Concentrated Components

- "nice" will finally mean:
  - A distribution over integral sets S, s.t. $|S| \in \{ \lfloor y_{B} \rfloor, \lceil y_{B} \rceil, \lceil y_{B} \rceil + 1 \}$
  - Capacity blow up O(ϵ)
  - each solution serves all the demand locally

Distributions Over Local Solutions for Concentrated Components

How to round (sample from) these nice distributions?:

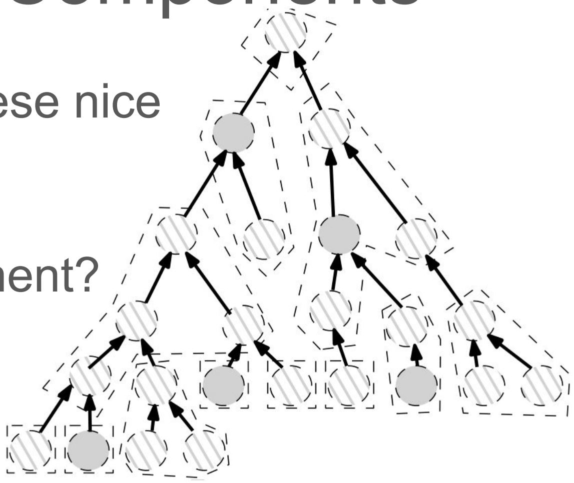


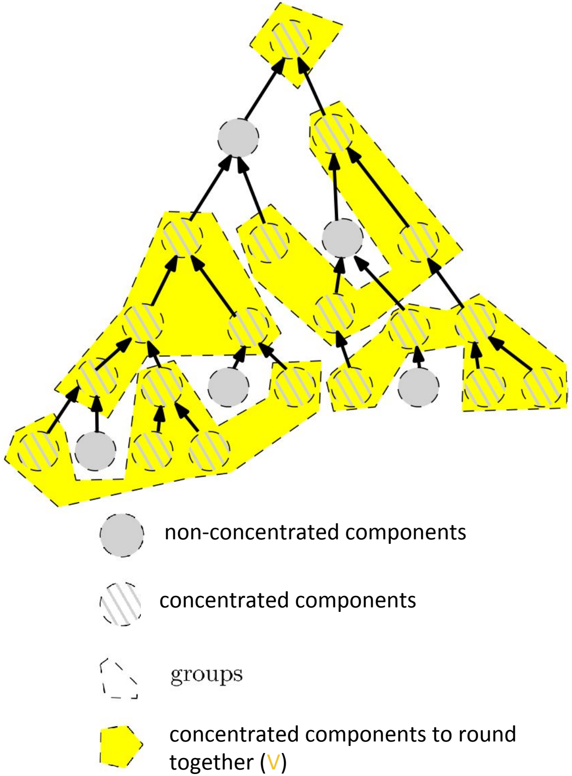
Distributions Over Local Solutions for Concentrated Components

How to round (sample from) these nice distributions?:

Independently for each component?

- Too many open facilities





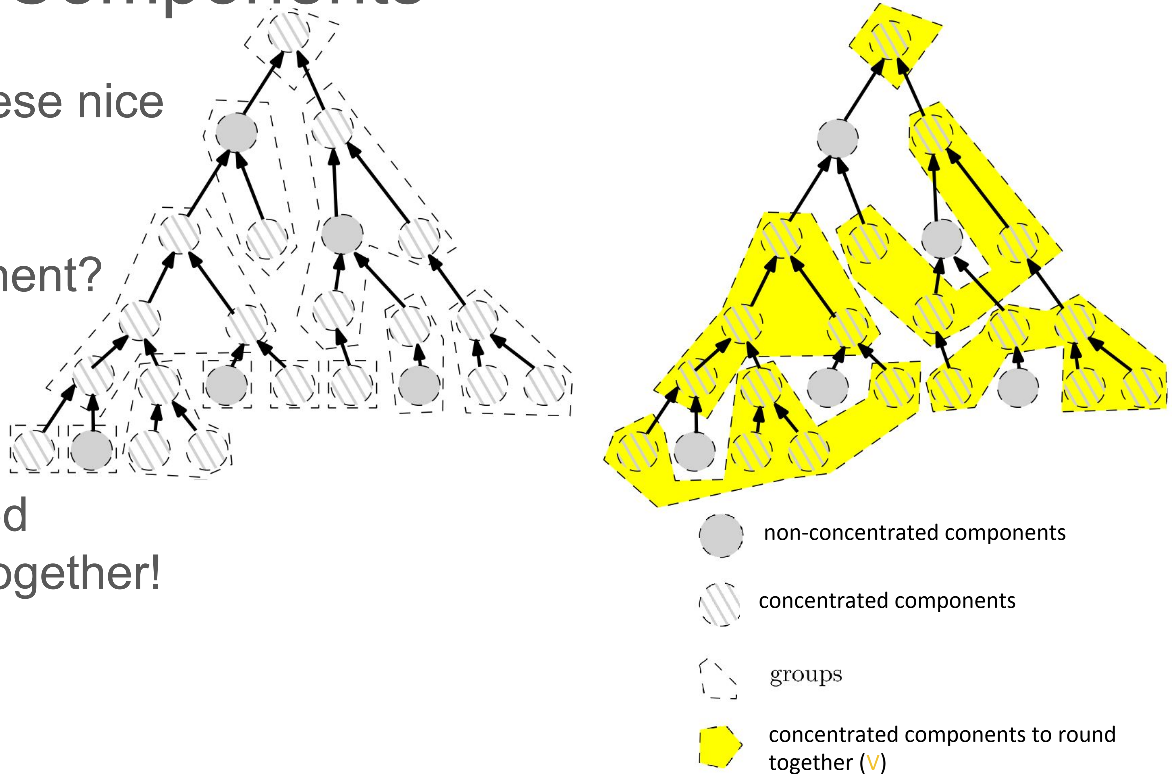
Distributions Over Local Solutions for Concentrated Components

How to round (sample from) these nice distributions?:

~~Independently for each component?~~
- ~~Too many open facilities~~

Dependently for all concentrated components in sibling groups together!

- O(1) total extra open facilities



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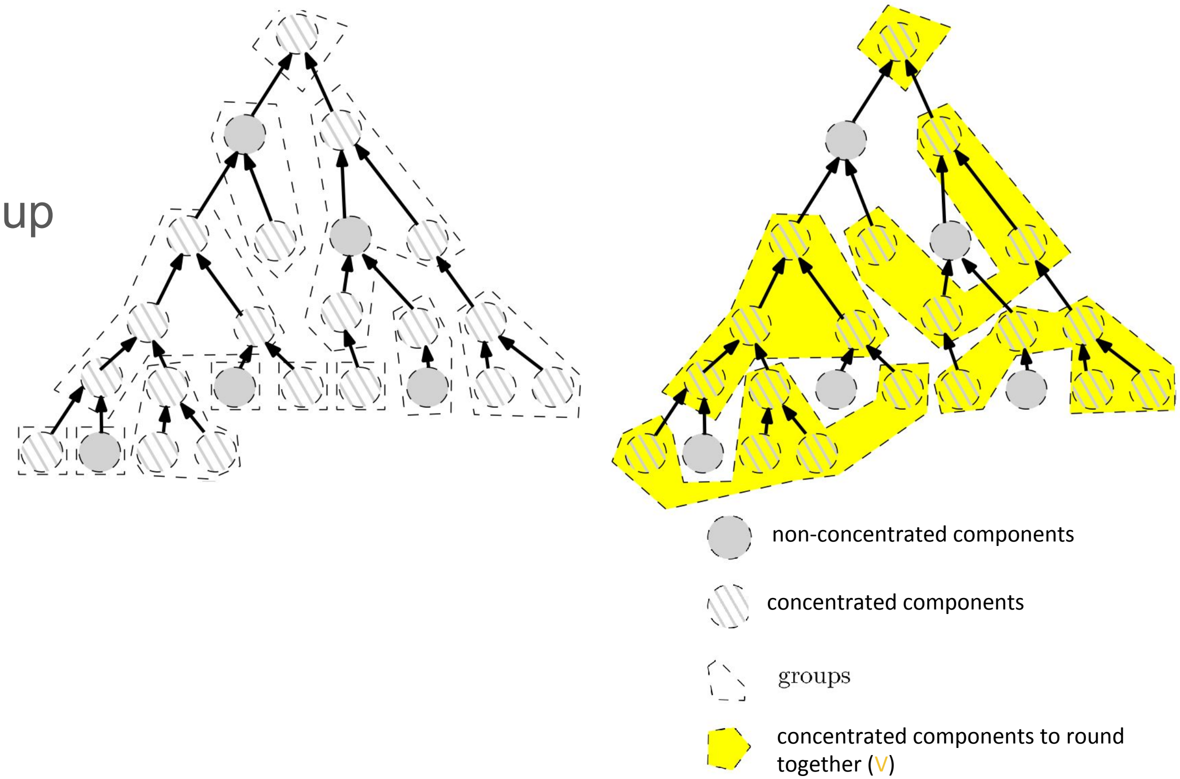
Putting it all together

For each group G,

- We may be opening O(1) extra facilities in all the children of a group



Putting it all together

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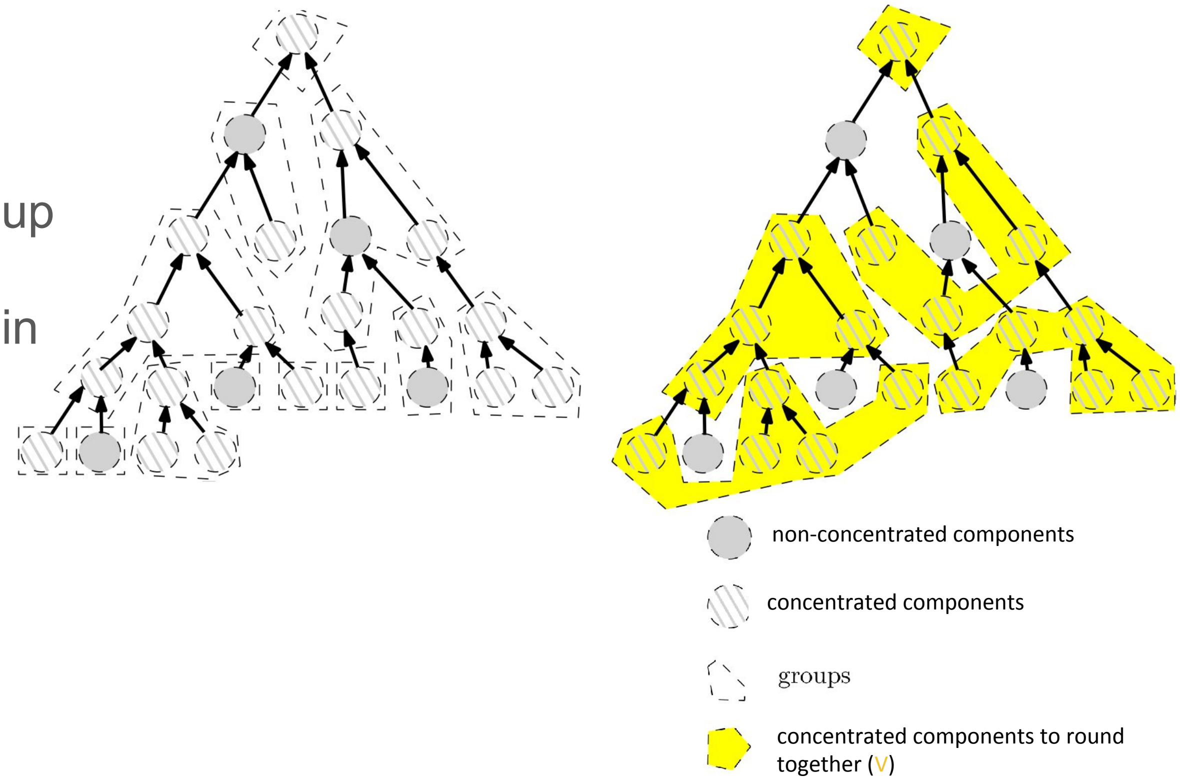
- Shut down O(1) facilities in G or in children.

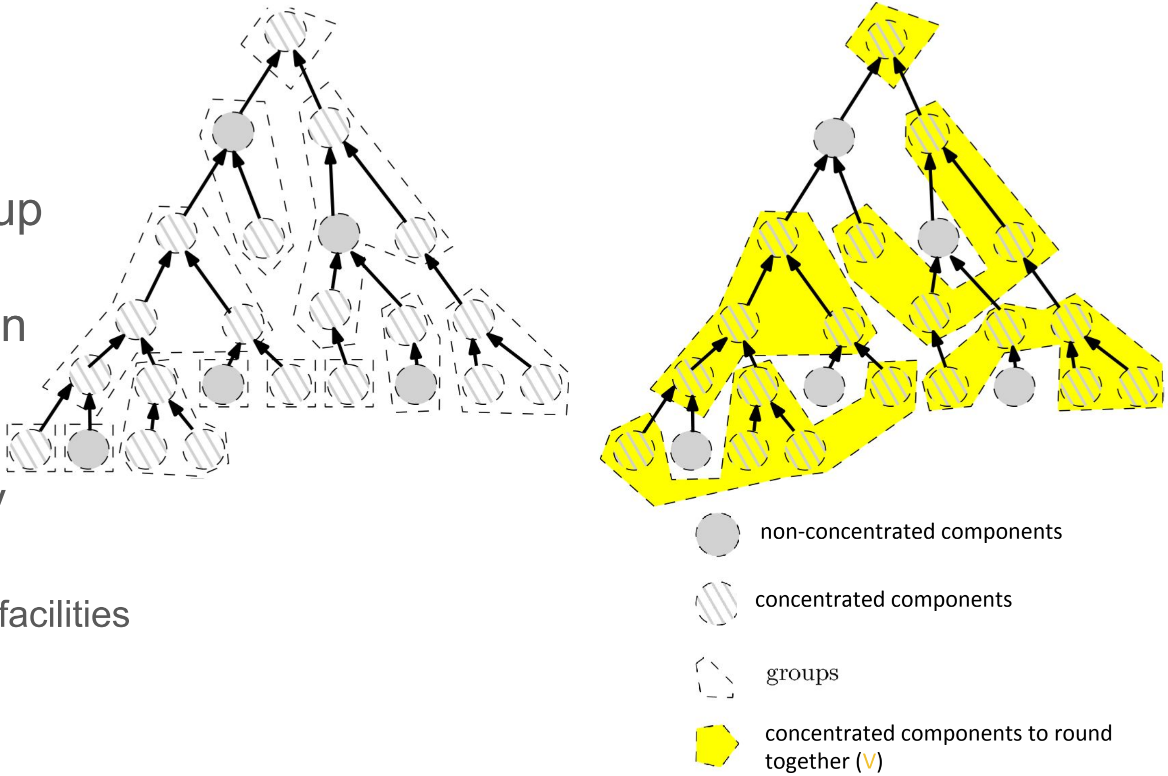


Putting it all together

For each group G,

- We may be opening O(1) extra facilities in all the children of a group
- Shut down O(1) facilities in G or in children.
- Serve their demand with capacity blow-up

A group has $\Omega(1/\epsilon)$ open facilities



Further research

- This finishes pseudo approximations for capacitated k-median.

- A true constant-factor approximation for capacitated k-median? (no violation)

- Configuration LP has big integrality gap!