On the Resiliency of Randomized Routing Against Multiple Edge Failures

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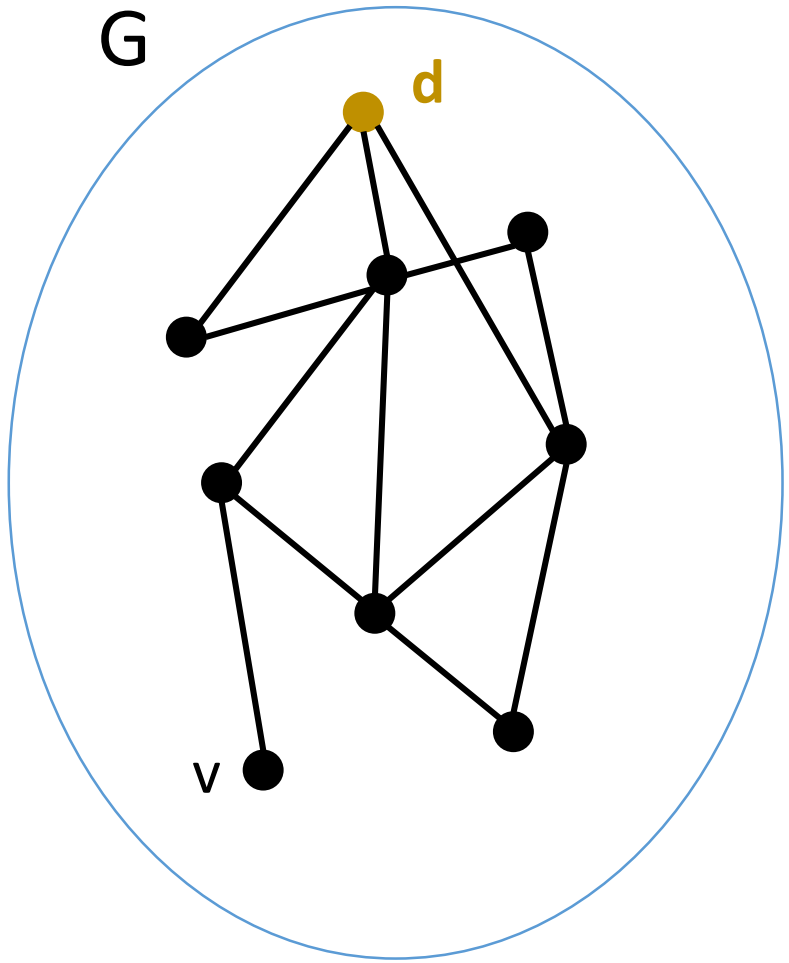
Network routing

Given:

- a network *G = (V, E)*
- a target vertex **d** of *V*

Goal: deliver a packet from *v* to **d**



Network routing – Many different settings

1. per-destination
2. per-incoming-port
3. per-source-destination
4. packet-header rewriting
5. packet duplication
6. dynamic
7. …

Two important properties:

- being resilient, and
- fast computation (static and local routing scheme)

Network routing – The problem we study

Given:

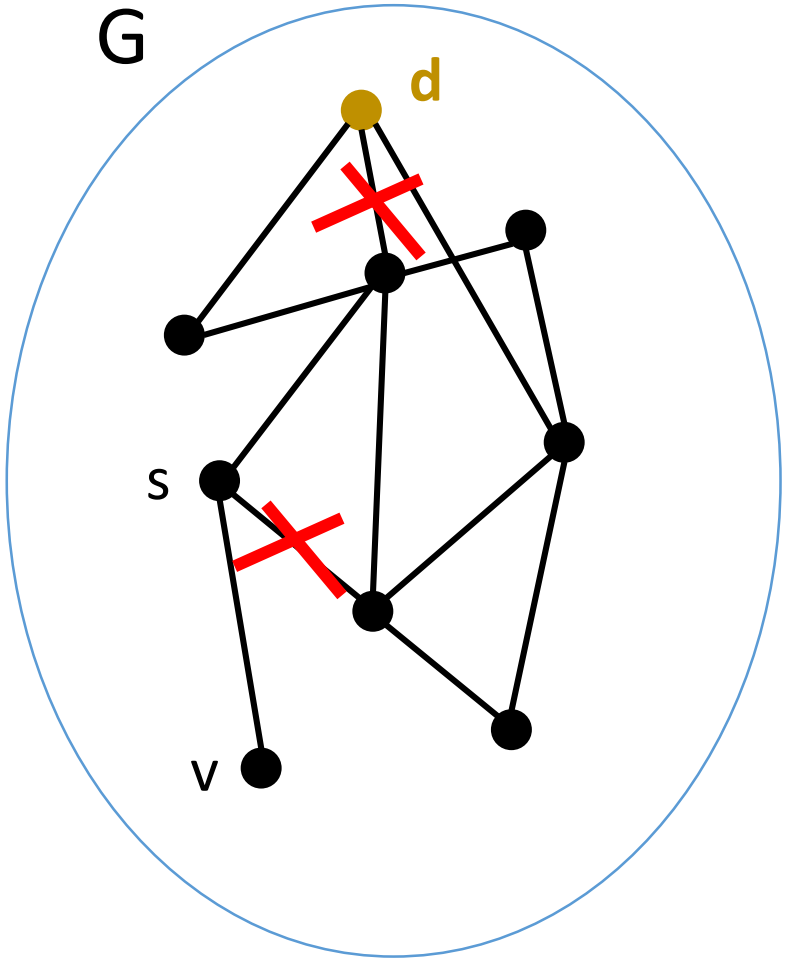
- a network *G = (V, E)*, a target vertex **d**
- a parameter *c*

Goal: Find a per-destination *static routing scheme* that delivers a packet from any source *s* to **d**

subject to: at most *c* links of *G* are failed

with property: routing is local (no packet-header rewriting; no broadcasting)

We say such scheme is *c-resilient*.

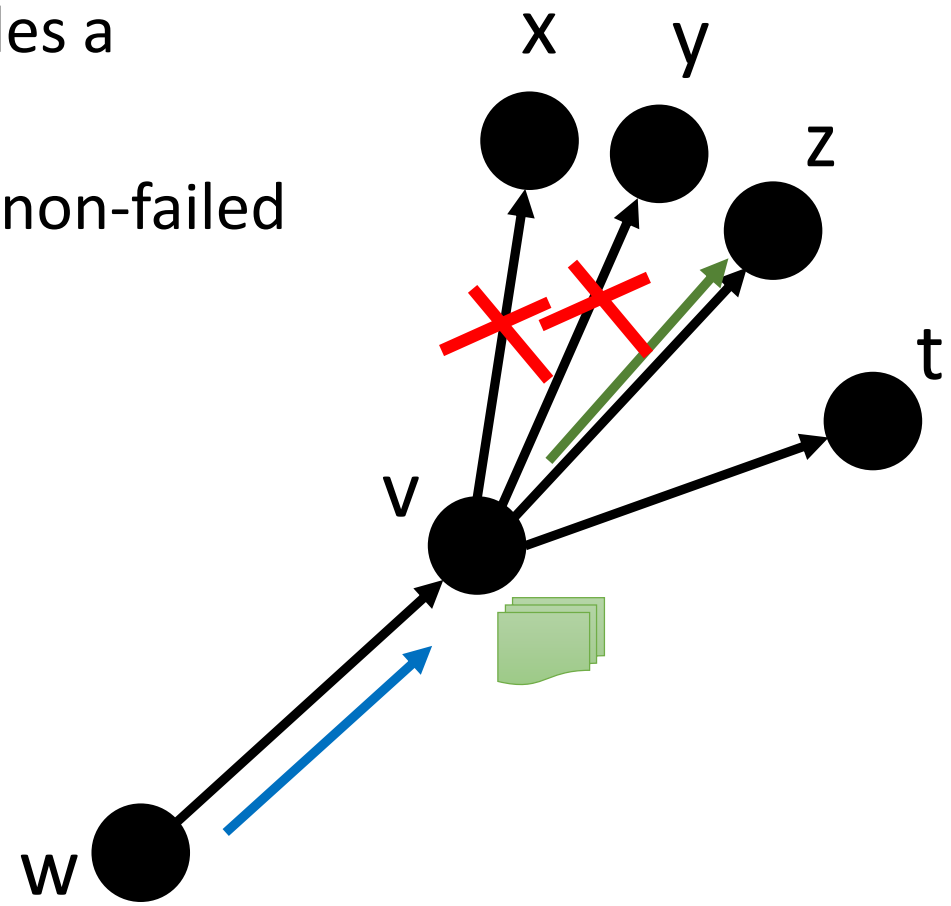


Local routing decisions

- Each vertex *v* has a list for each incoming link
- For incoming link from *w*, table provides a permutation of outgoing links
- The routing is continued through first non-failed link in the list

v:

x:	y	z	t	
y:	t	x	w	z
w:	x	y	z	t
z:	t	z	x	
t:	w			



Example

s:

start:	u

u:

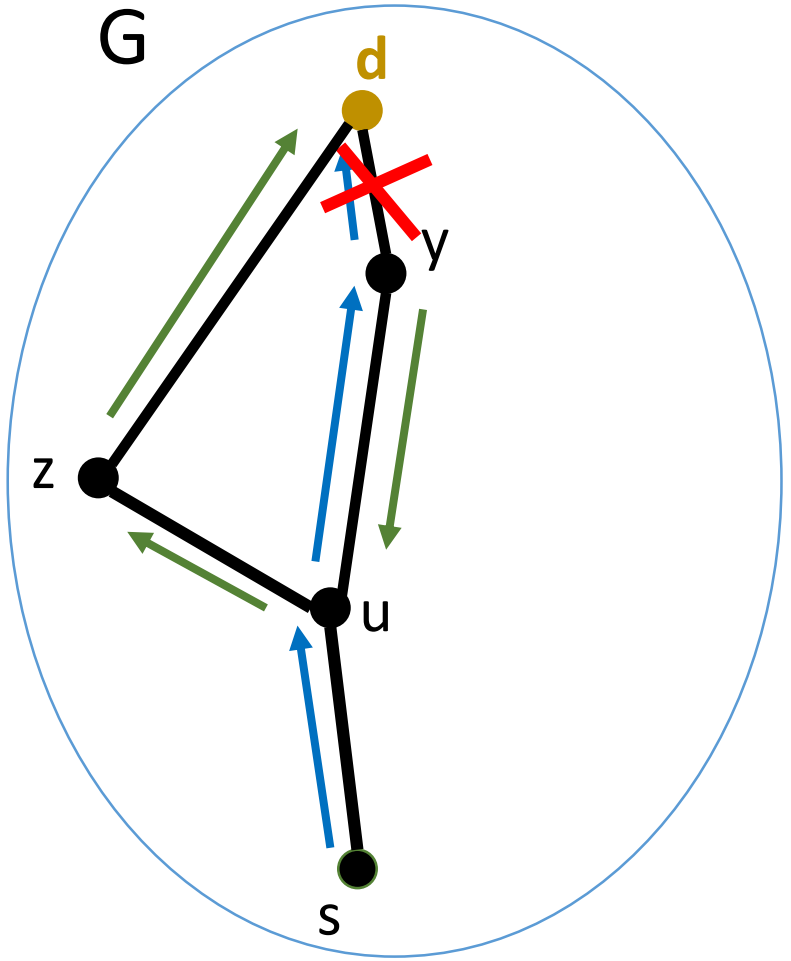
s:	y
y:	z

y:

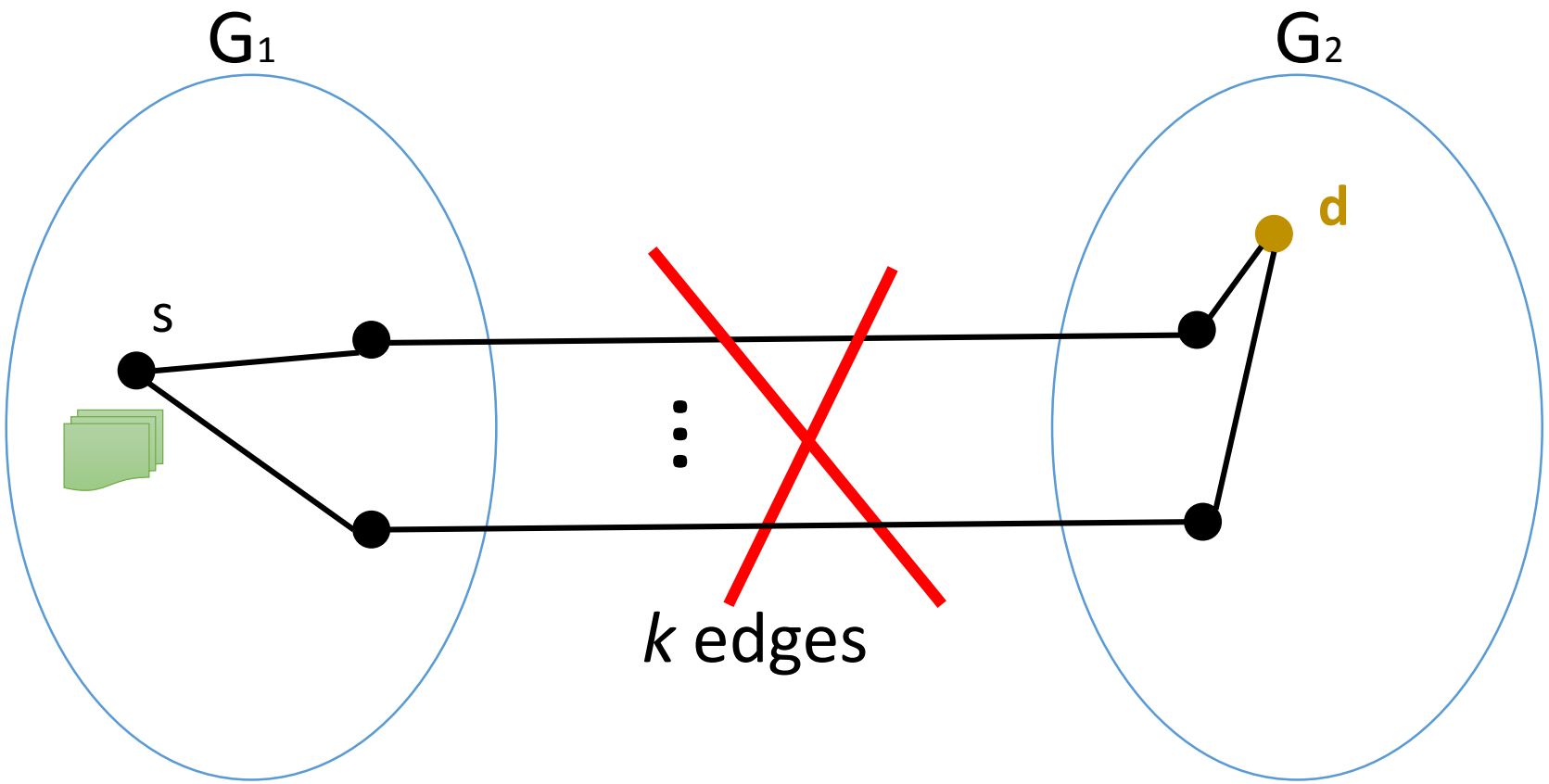
u:	d	u

z:

u:	d



We need *k*-connectivity



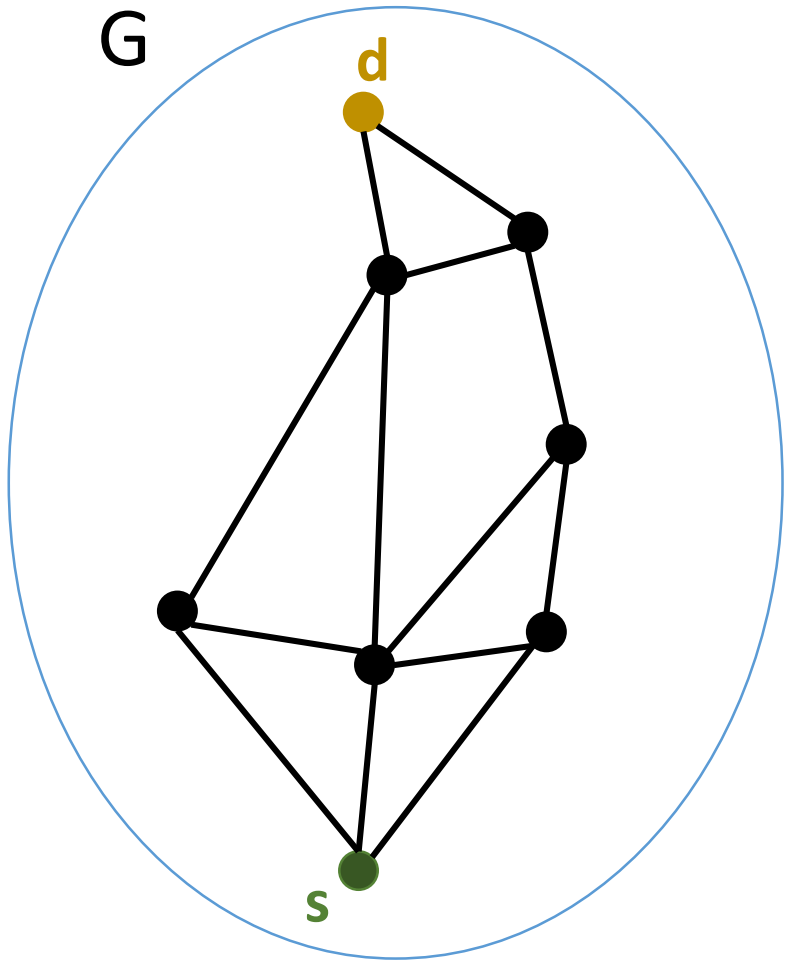
Necessary condition: *c*-resilience requires $k > c$.

Big challenge: Is $k > c$ sufficient too?

(from now, assume c < k)

Attempt 1 – Route along edge-disjoint paths

- [*Menger's theorem*] Between any u and v of G, there are at least k-edge disjoint paths.

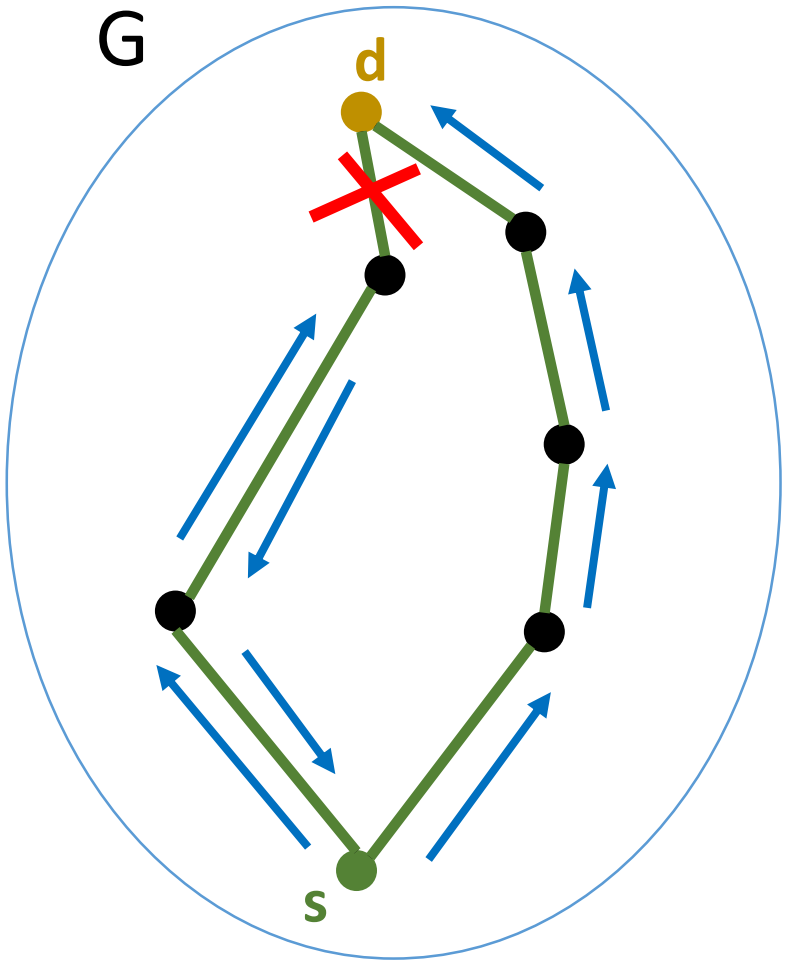


Attempt 1 – Route along edge-disjoint paths

- [*Menger's theorem*] Between any u and v of G, there are at least k-edge <u>disjoint</u> paths.

1. Find path packing
2. Route along paths
3. On failed edge, retreat back to s, and choose the next path

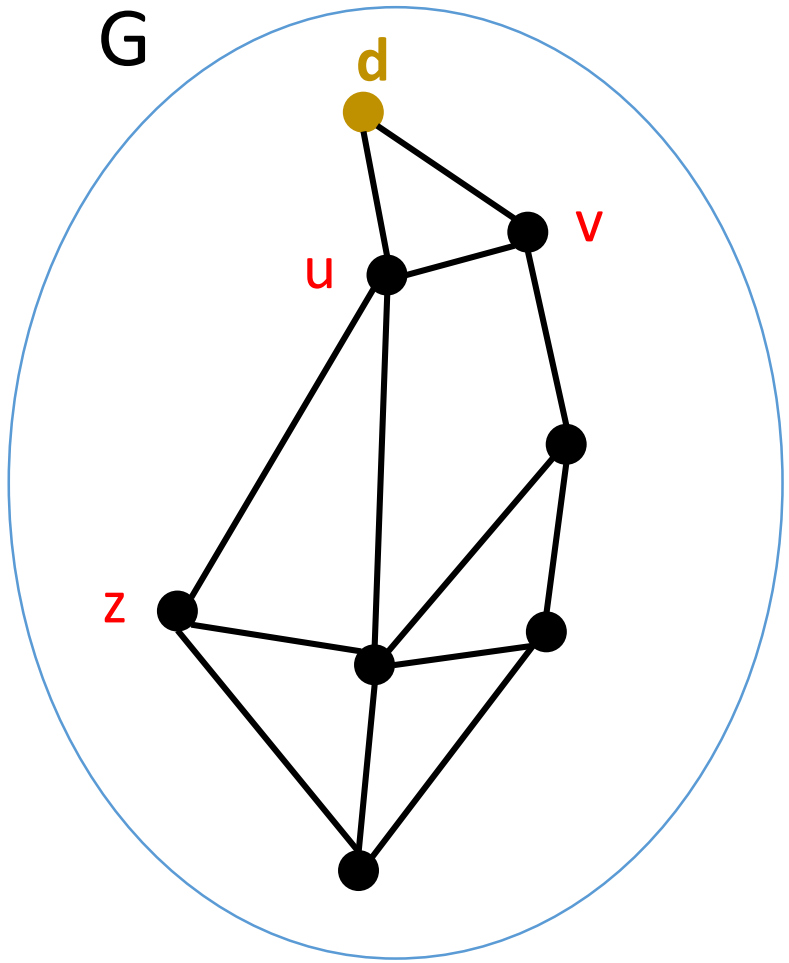
- But, <u>this is not satisfactory</u>, because the table is source-**d** based.



Attempt 2 – Packing arborescences

[*Edmonds, 1973*] A k-edge-connected graph G contains k arc-disjoint d-rooted arborescences.

1. Find arborescences packing
2. Route along arborescences
3. On failed edge, choose the next available arborescence (*circular routing*)



[EGR-INFOCOM14]
[CNMPGSS-INFOCOM16]

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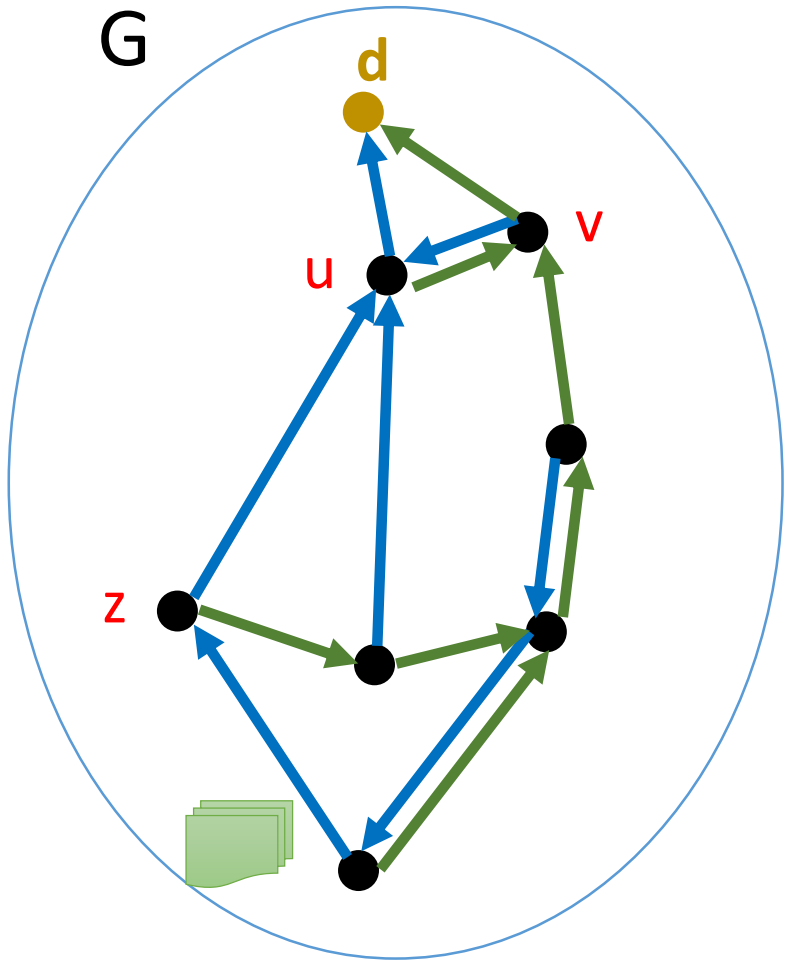

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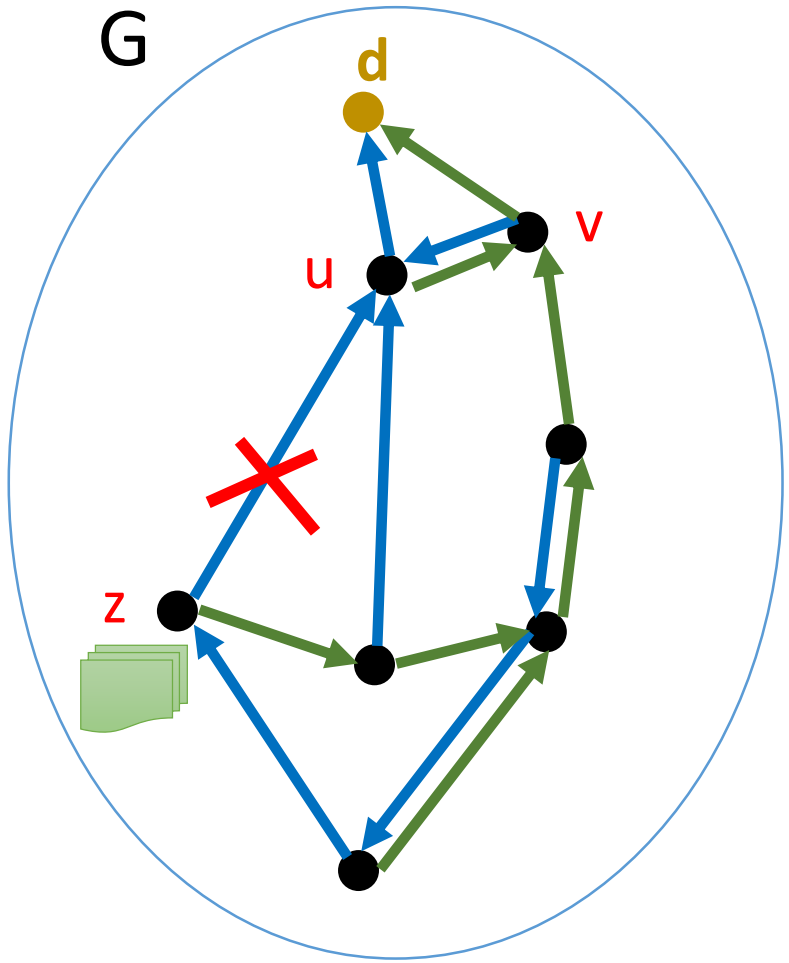

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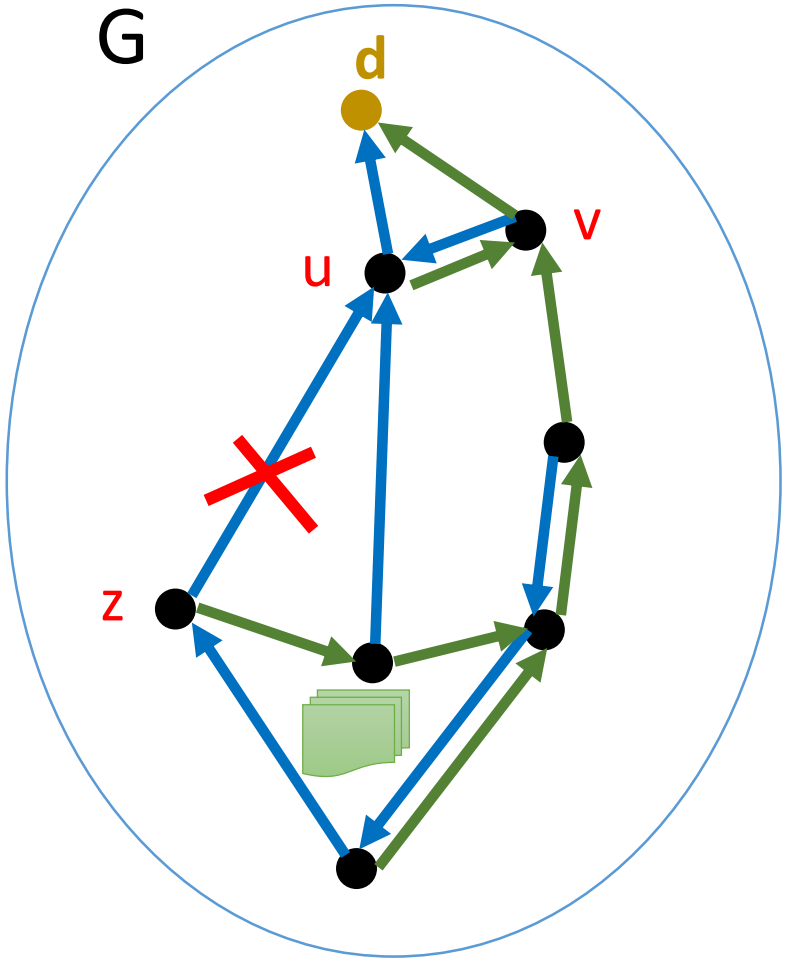

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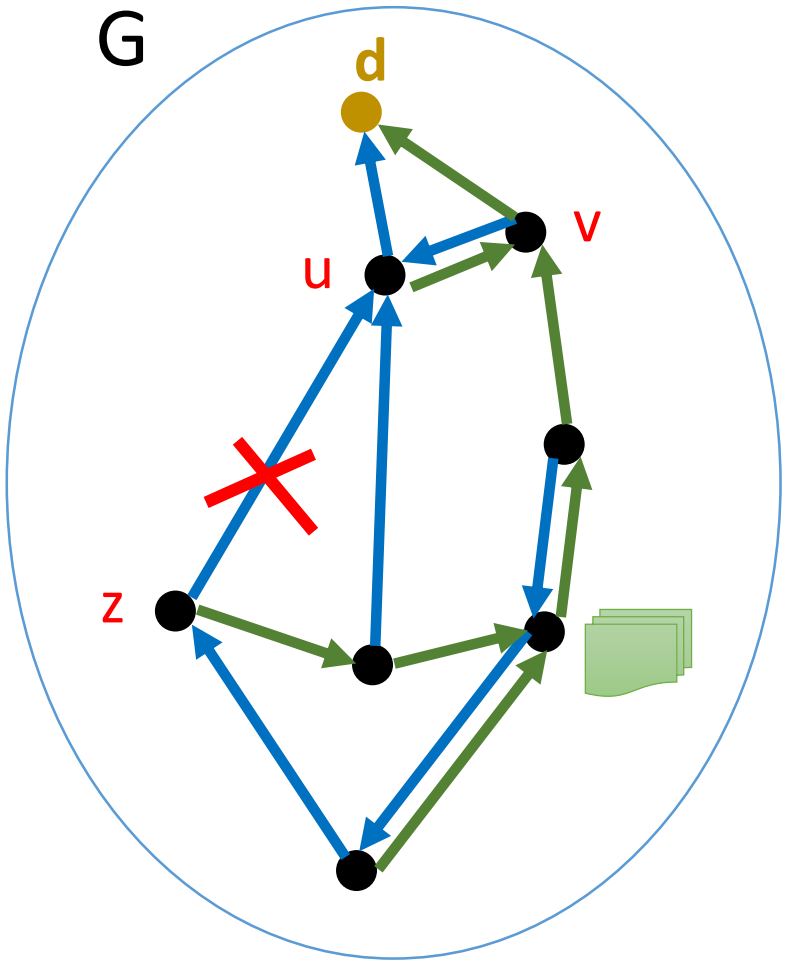

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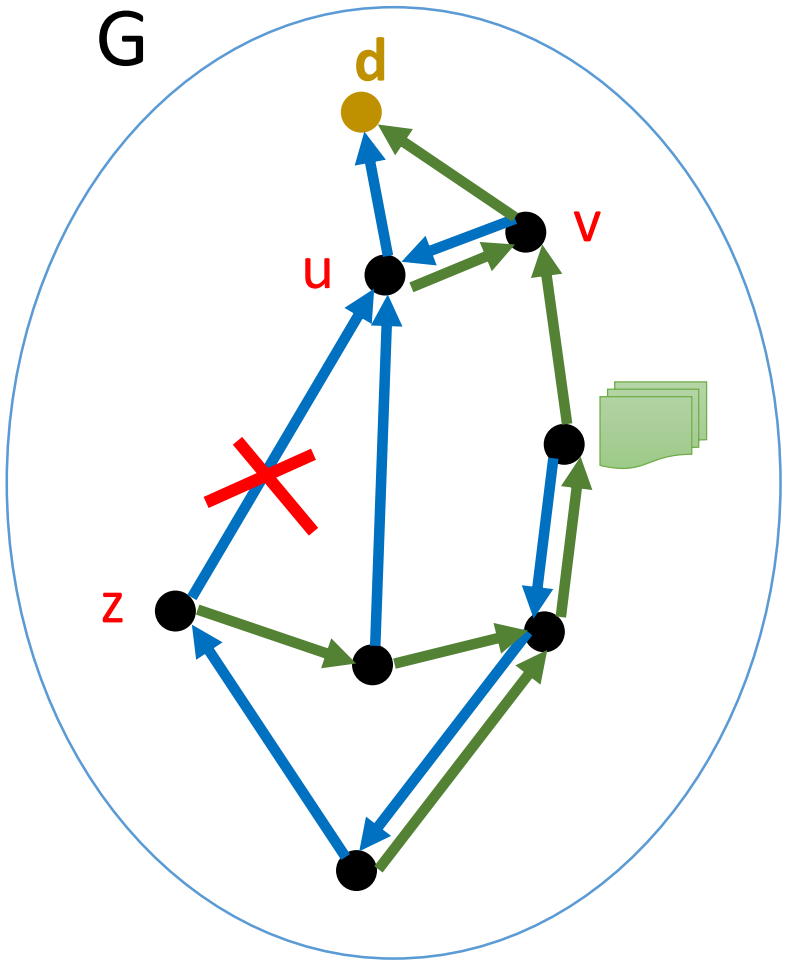

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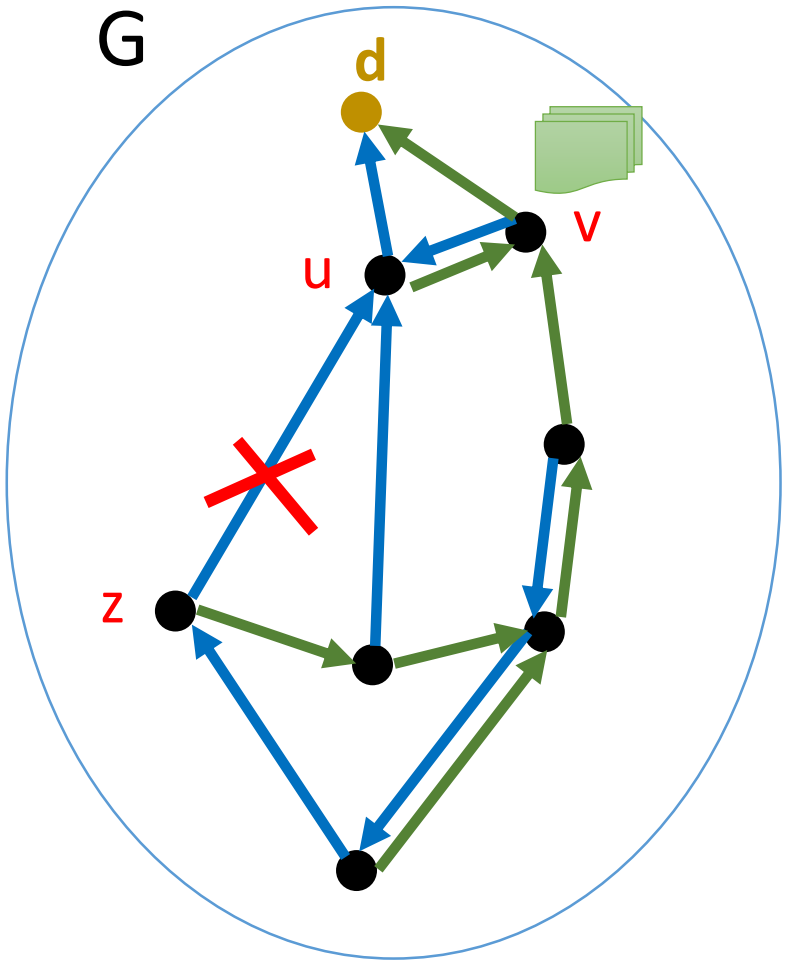

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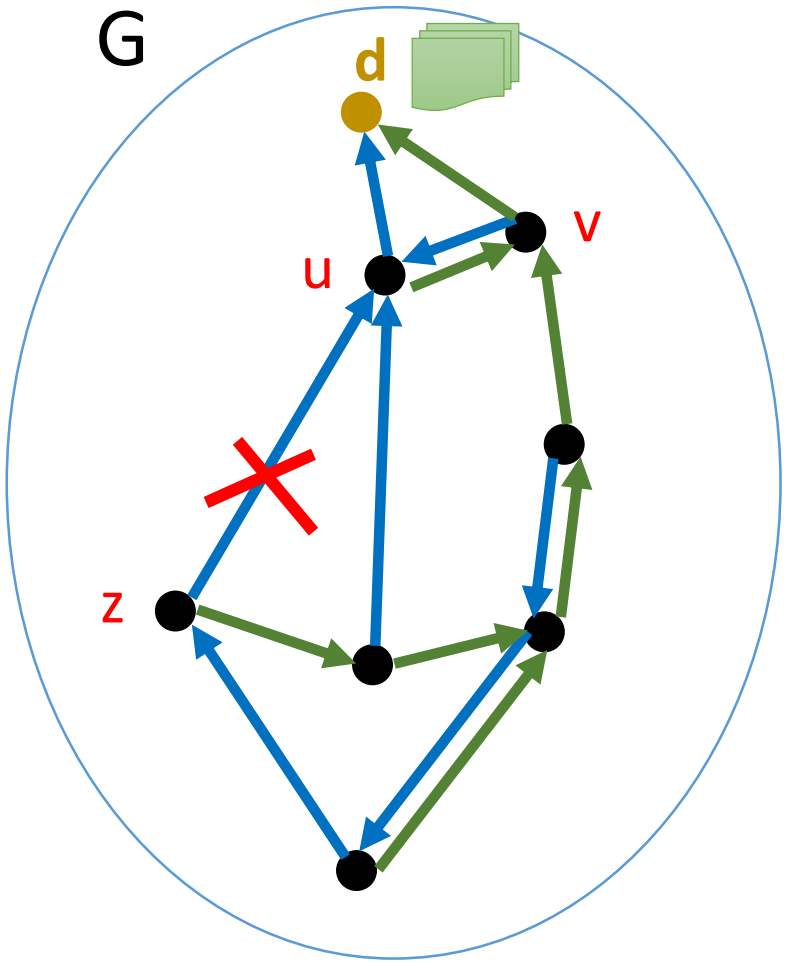
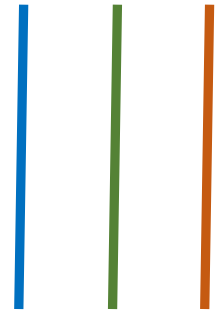
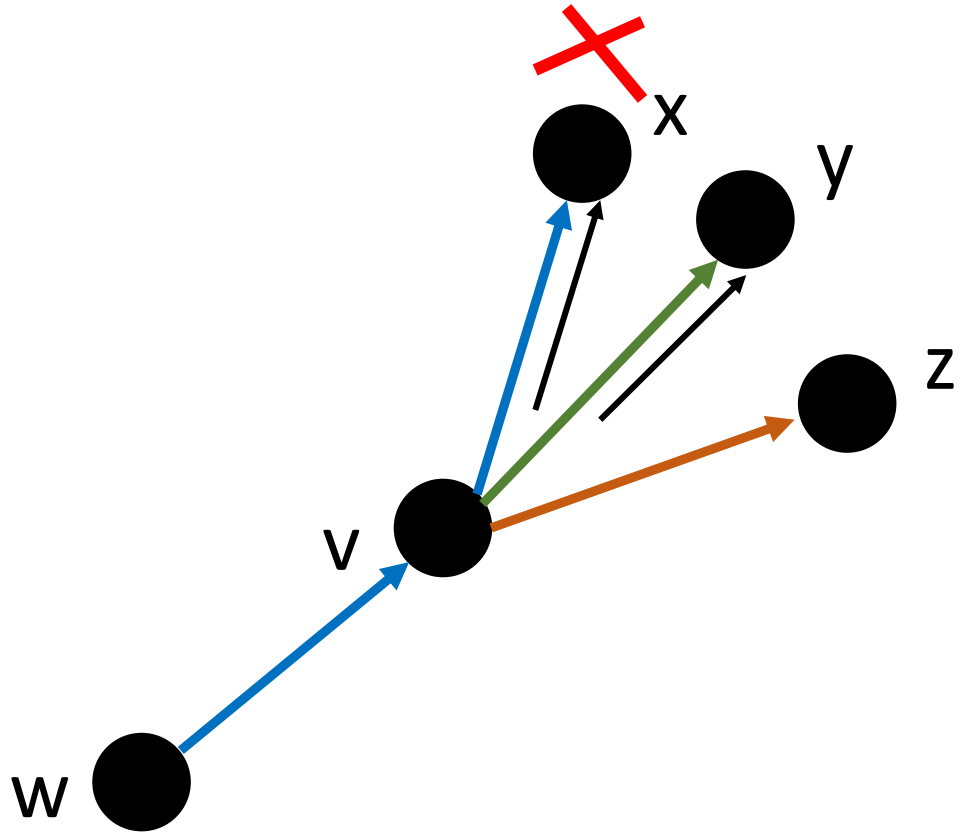


Arborescences give k/2 resilience

- Find an arborescence packing
- Order the arborescences



- Route along arborescences; on failed edge route along the next arborescence in the ordering



Each failure can affect two arborescences.

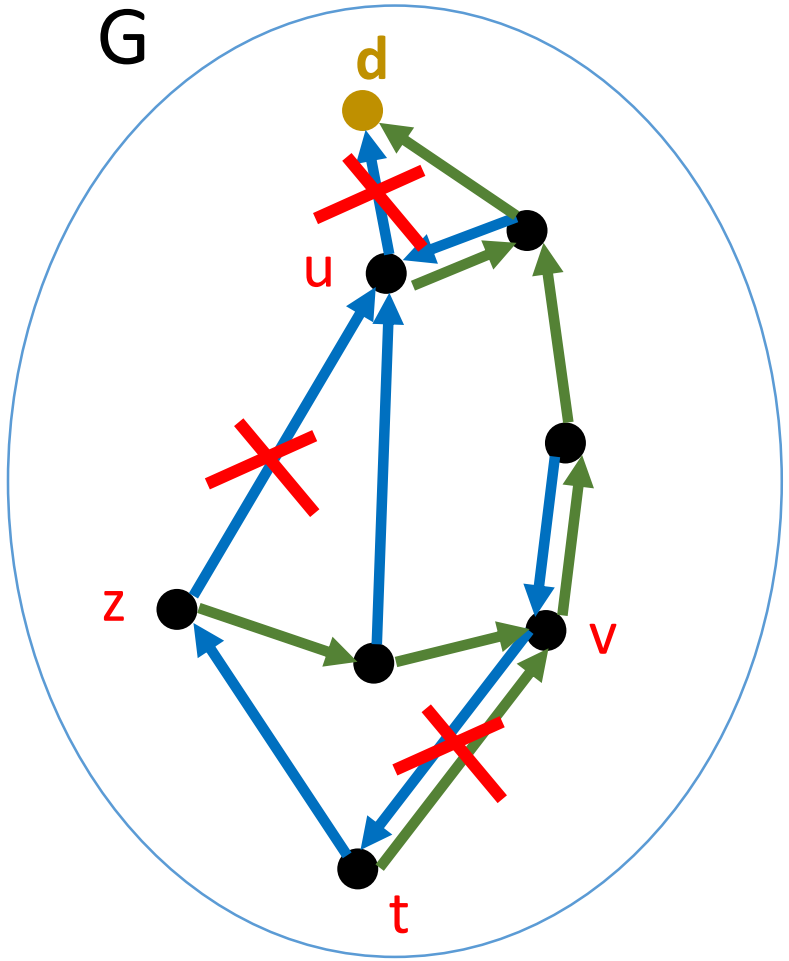
The example is tight (think of a cycle).

Can we do better?

Our result: We settle this challenge and, given a *k*-connected graph, provide a randomized algorithm that is *(k-1)*-resilient.

(Naive use of randomization would be to just take a random walk. But this would be very inefficient both in amount of randomness used and the expected length of the routing paths.)

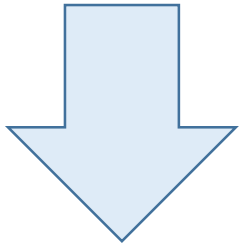
Two types of failed edges



- {t, v} edge is shared
- edges {z, u} and {u, d} are non-shared

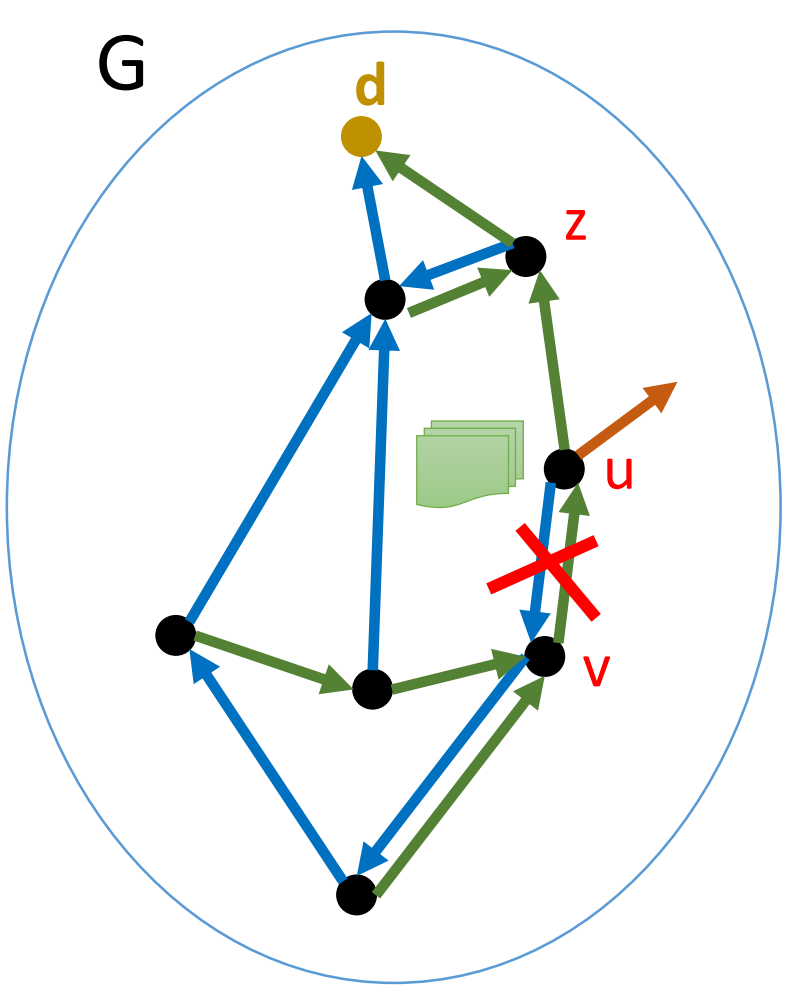
Non-shared failed edges are not a problem

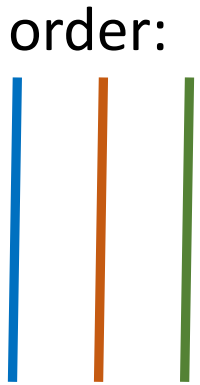
One failed edge destroys at most one arborescence



As there are at most k-1 failed edges, at least one arborescence has no failed arc.

How to "recycle" failed edges? Bounce!





- Route from u along blue.
- If (u, v) failed, no blue u-**d** path, but ...
- (u, v) is shared and there is a green u-**d** path!
- So, bounce at u.

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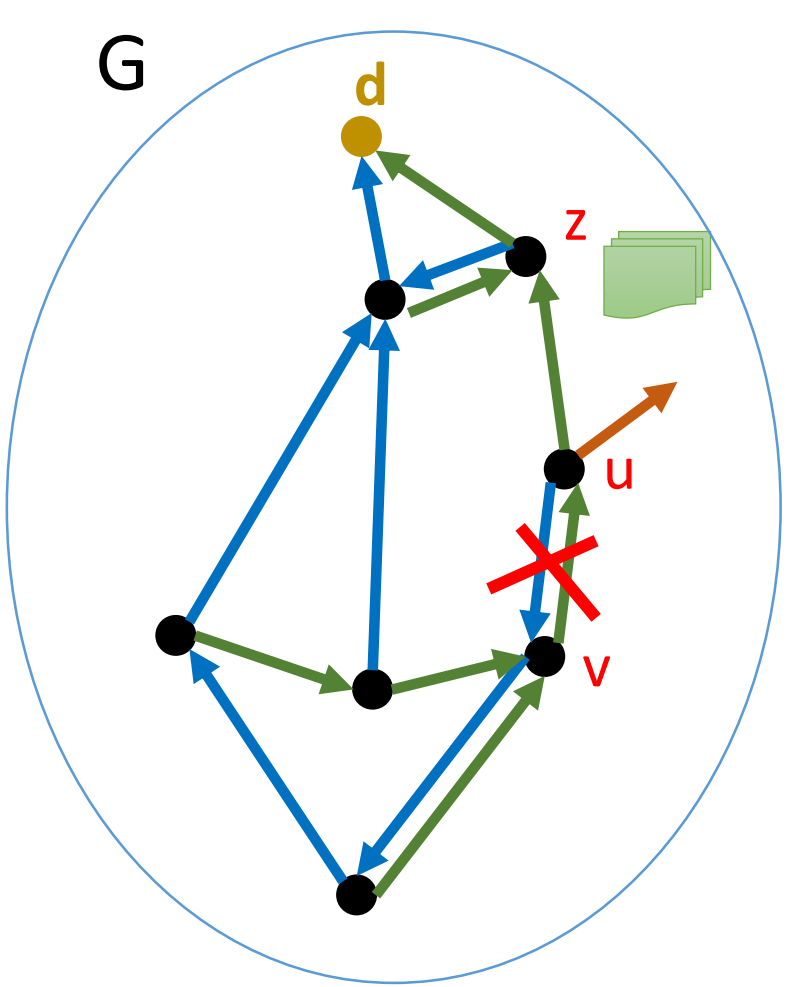

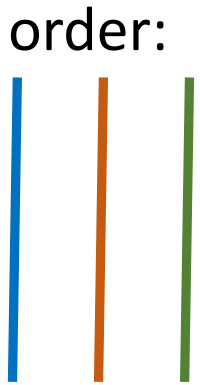

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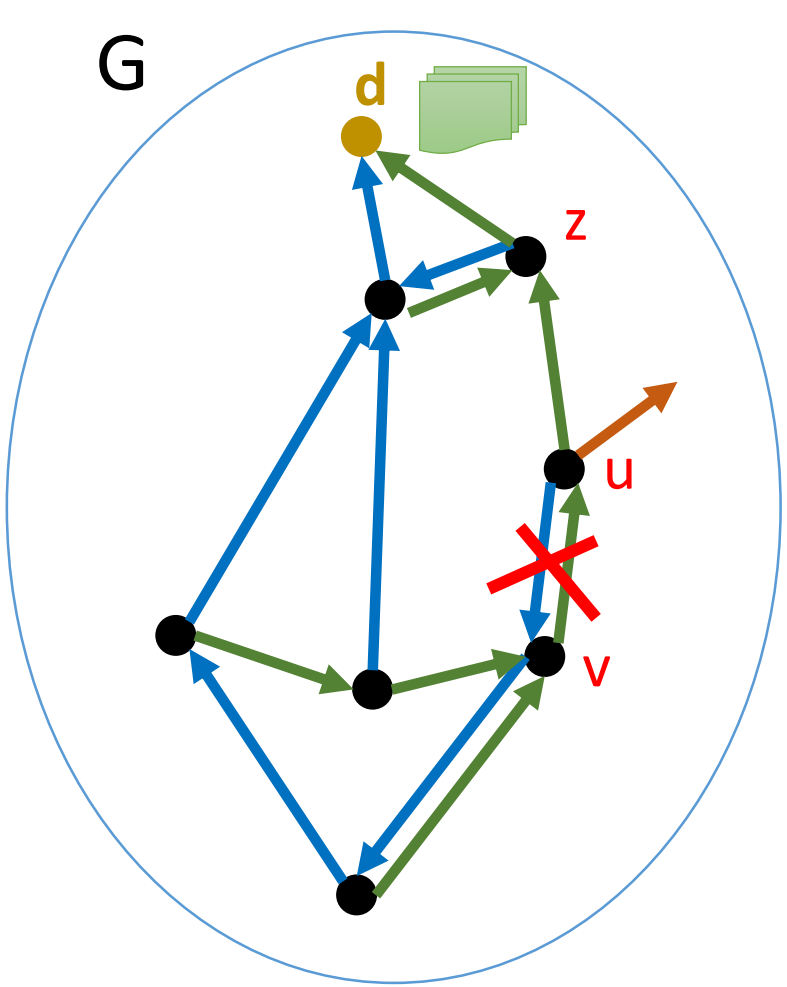

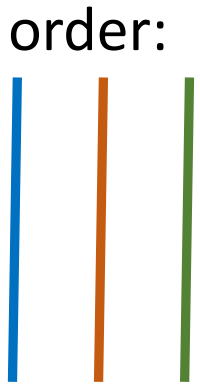

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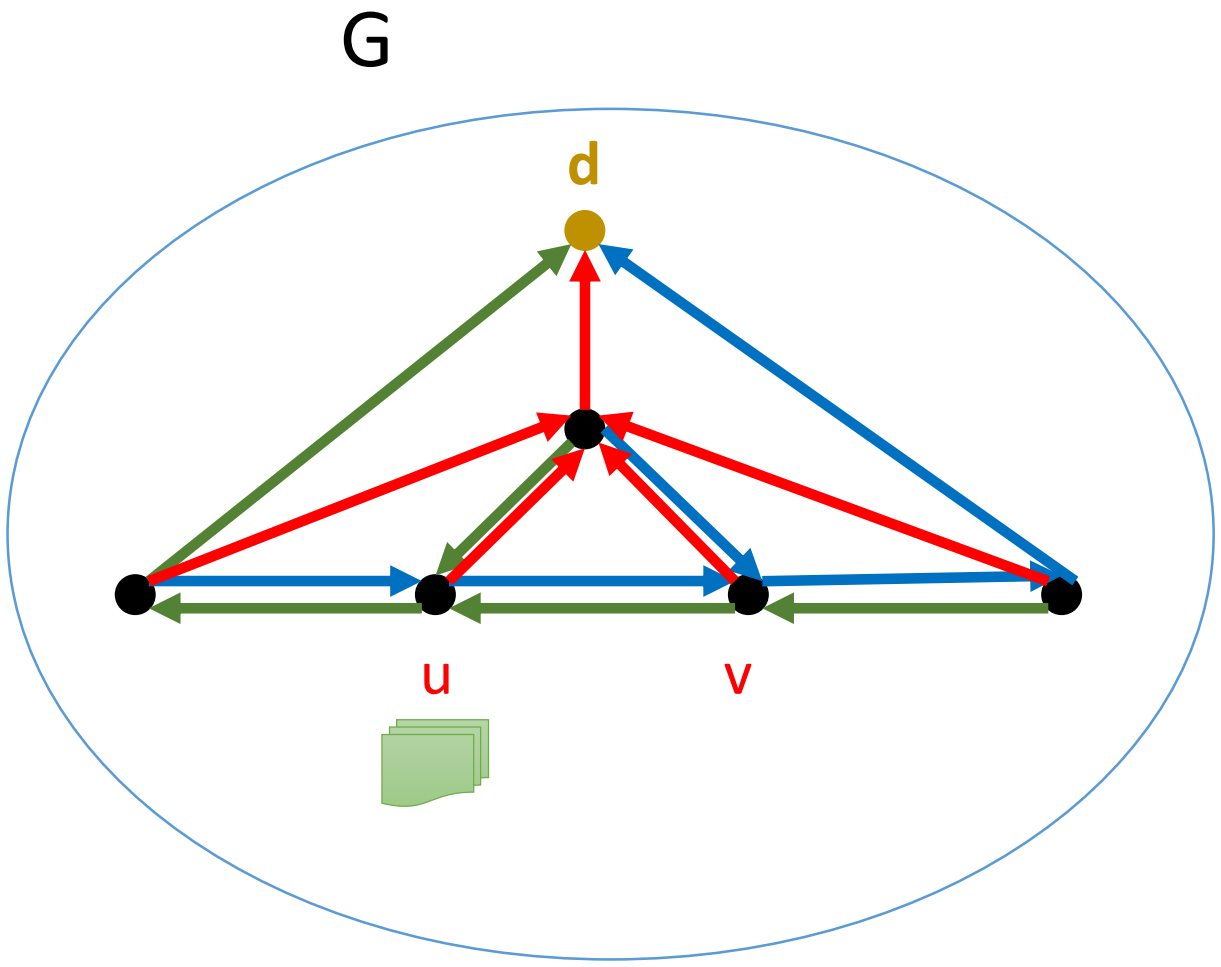
Bouncing every time does not work



1. The packet is on blue at u.

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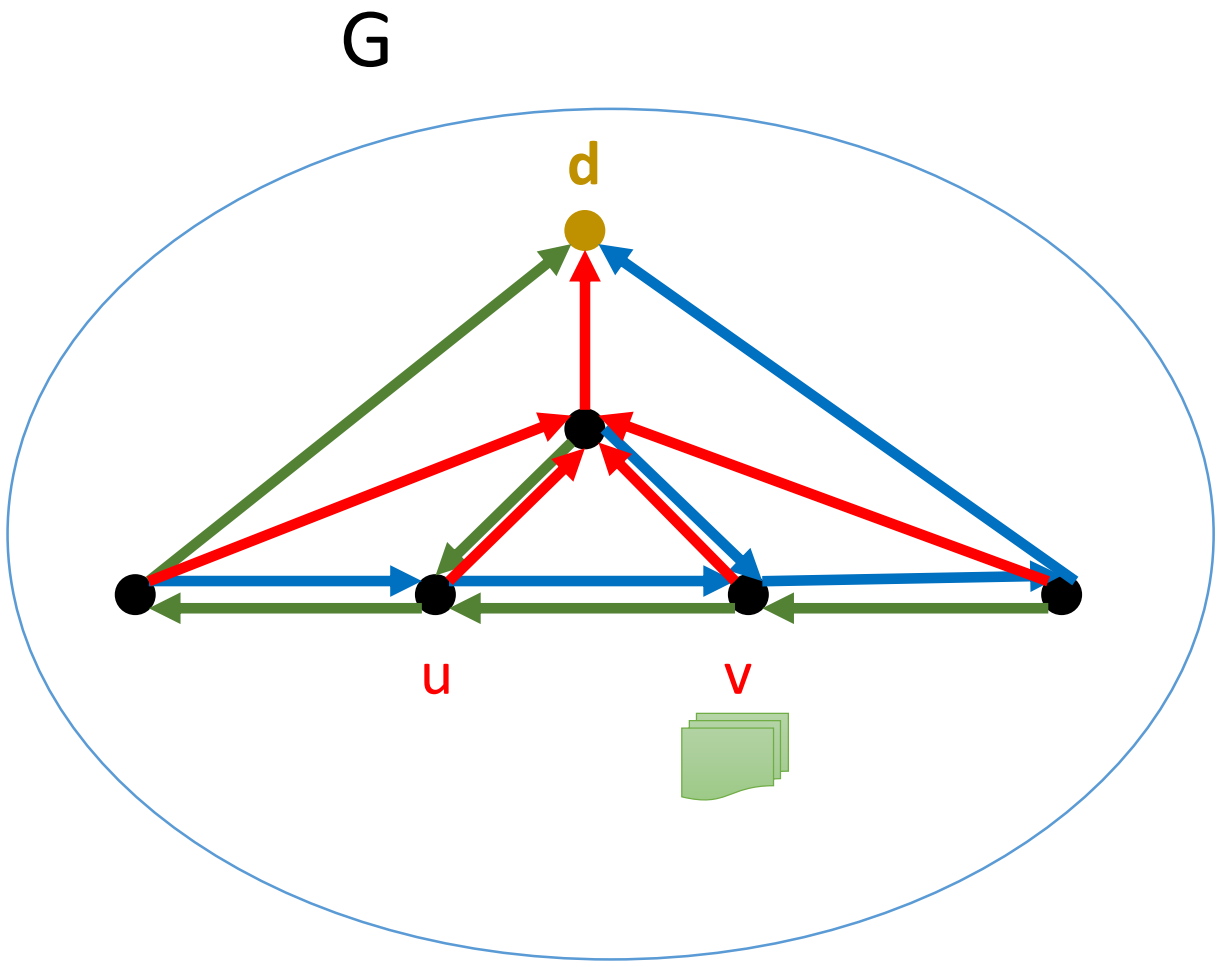

1. The packet is on blue at u.
2. The packet is on blue at v.

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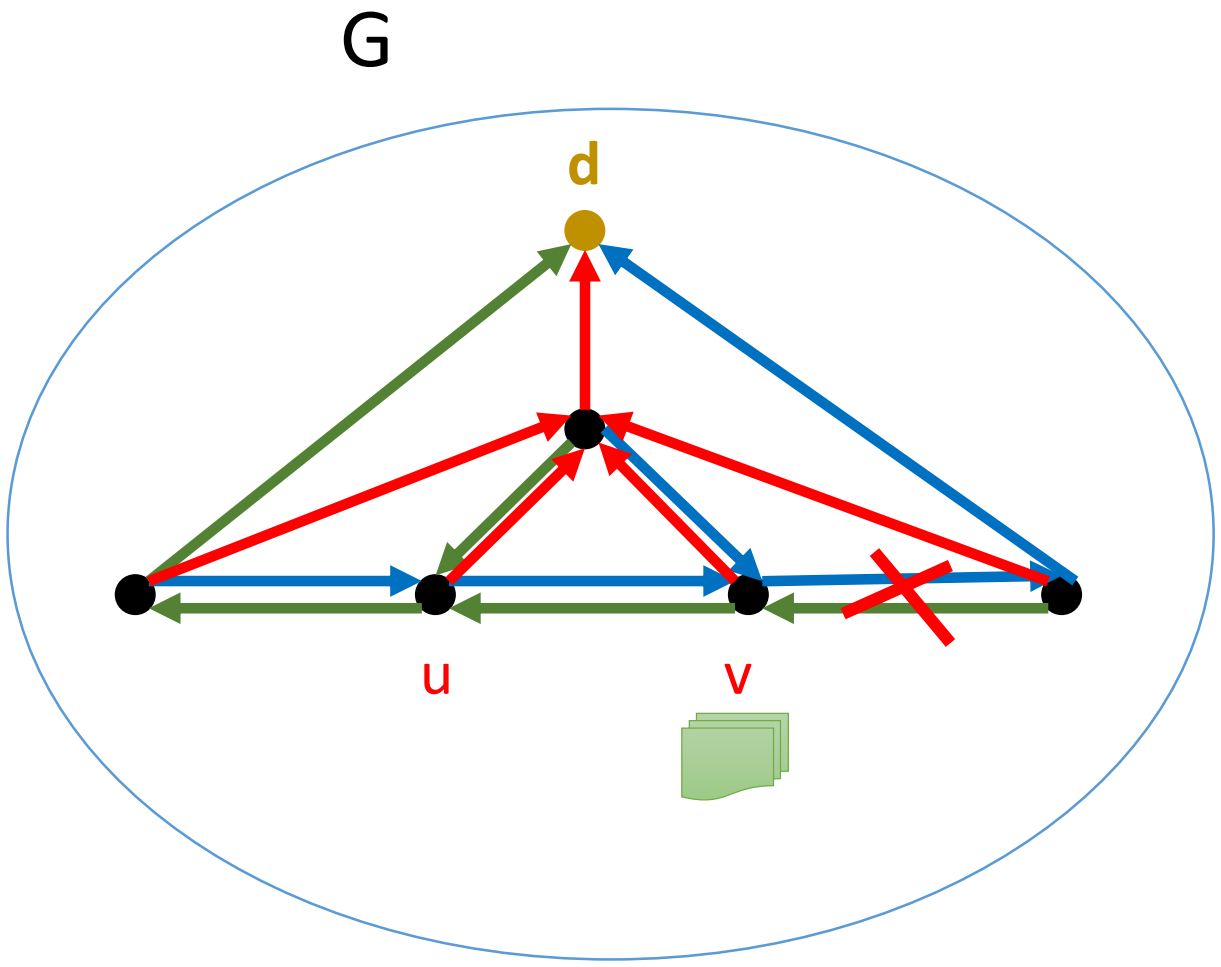

1. The packet is on blue at u.
2. The packet is on blue at v.
3. The next link failed. Bounce.

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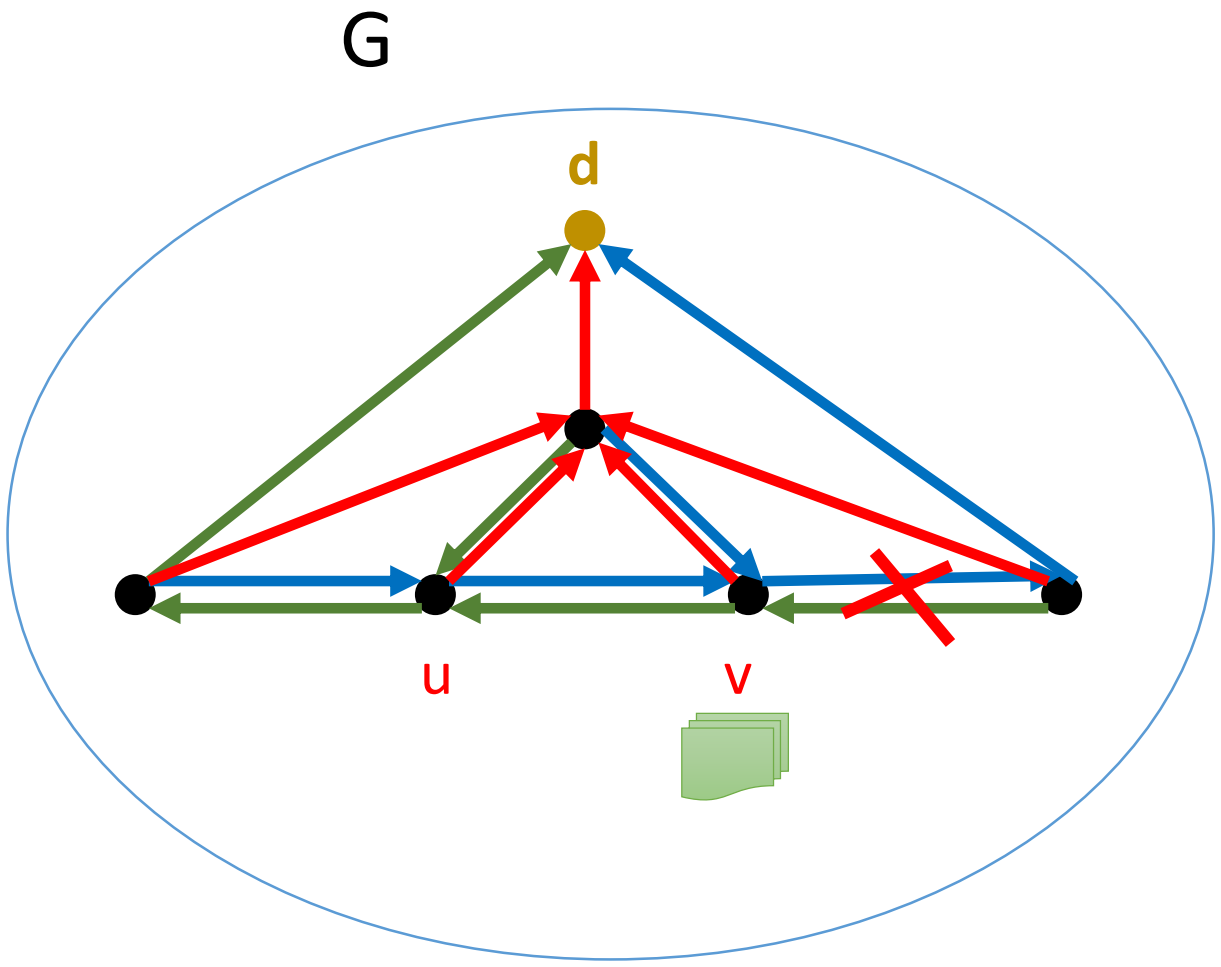

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3. The next link failed. Bounce.
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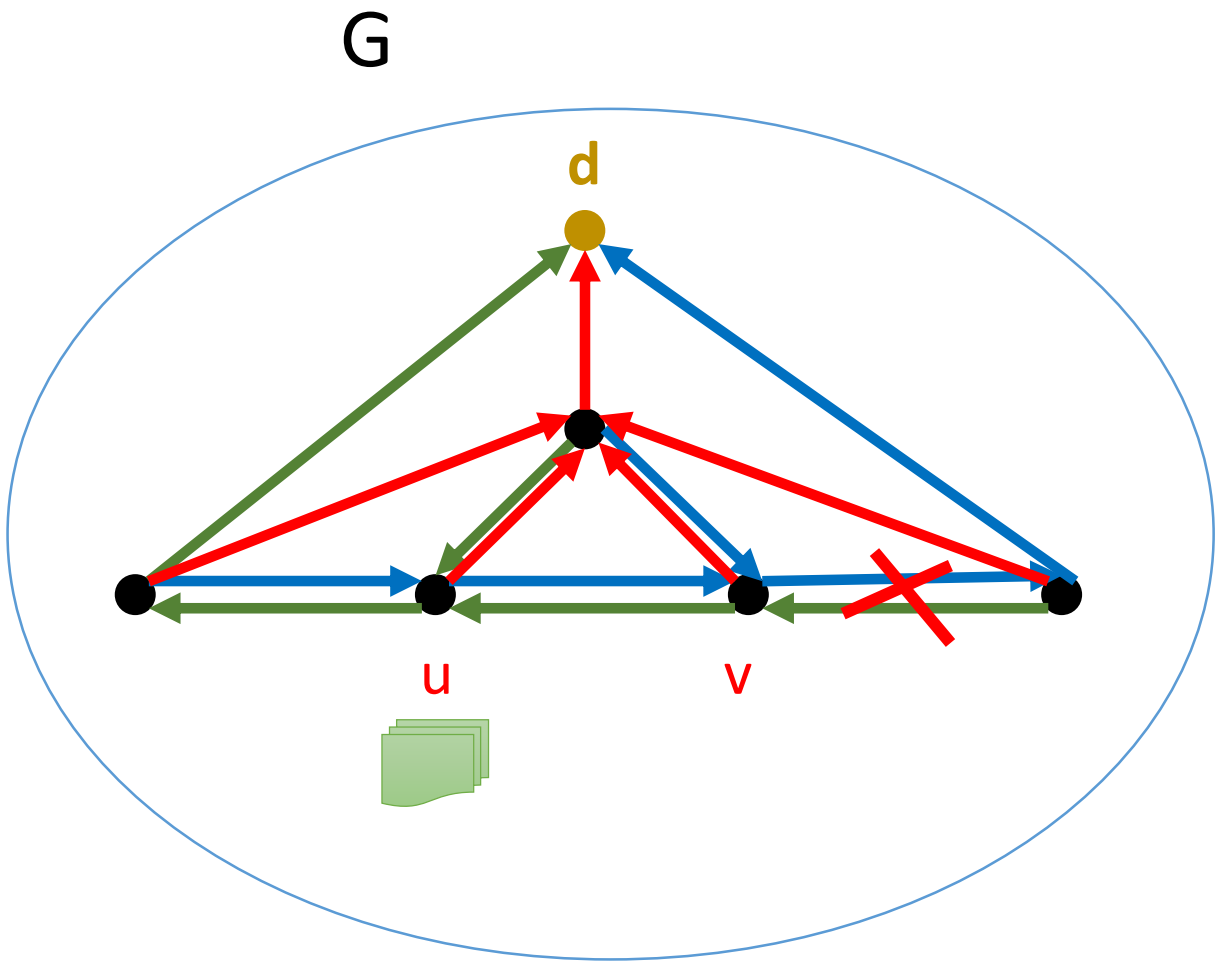

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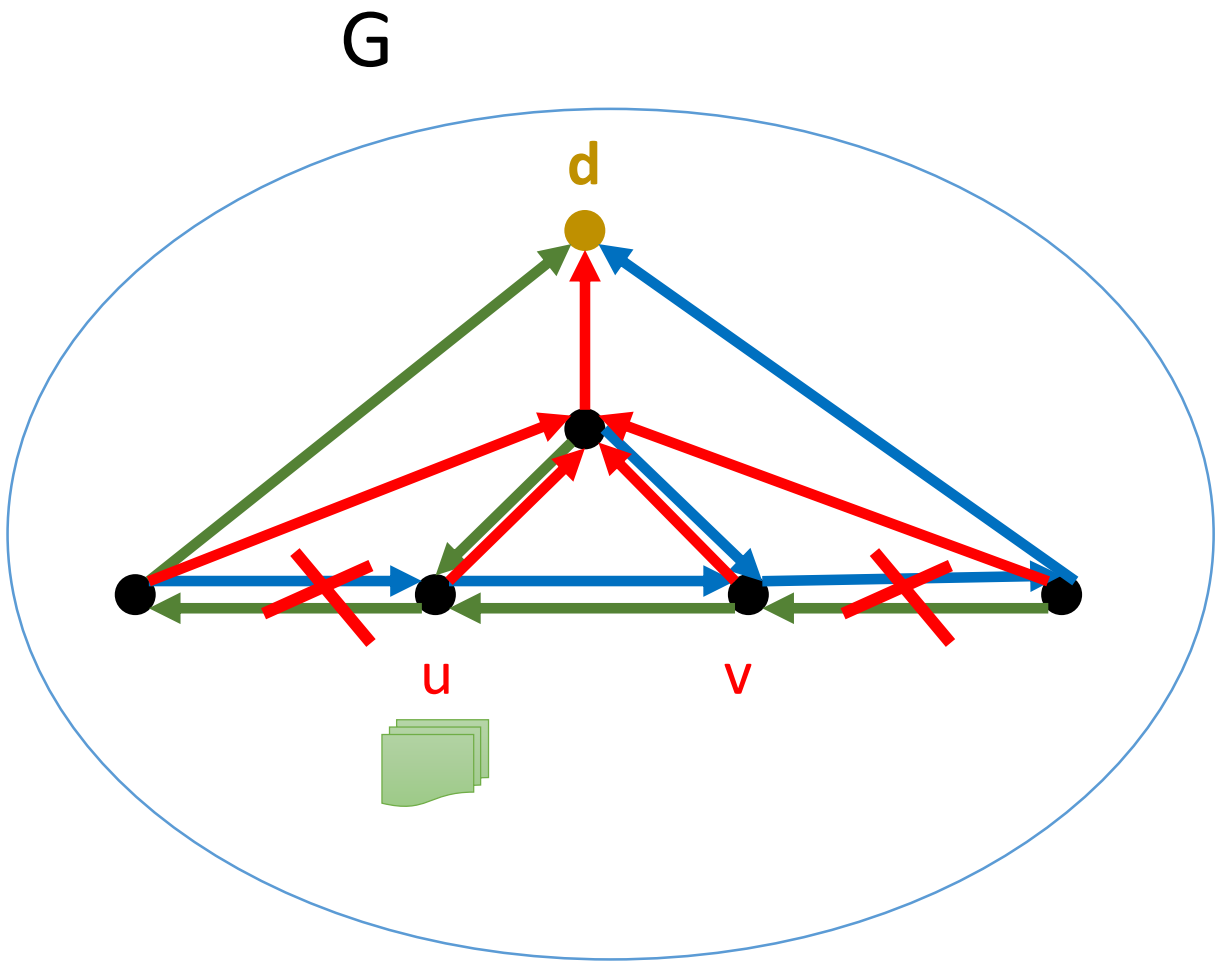

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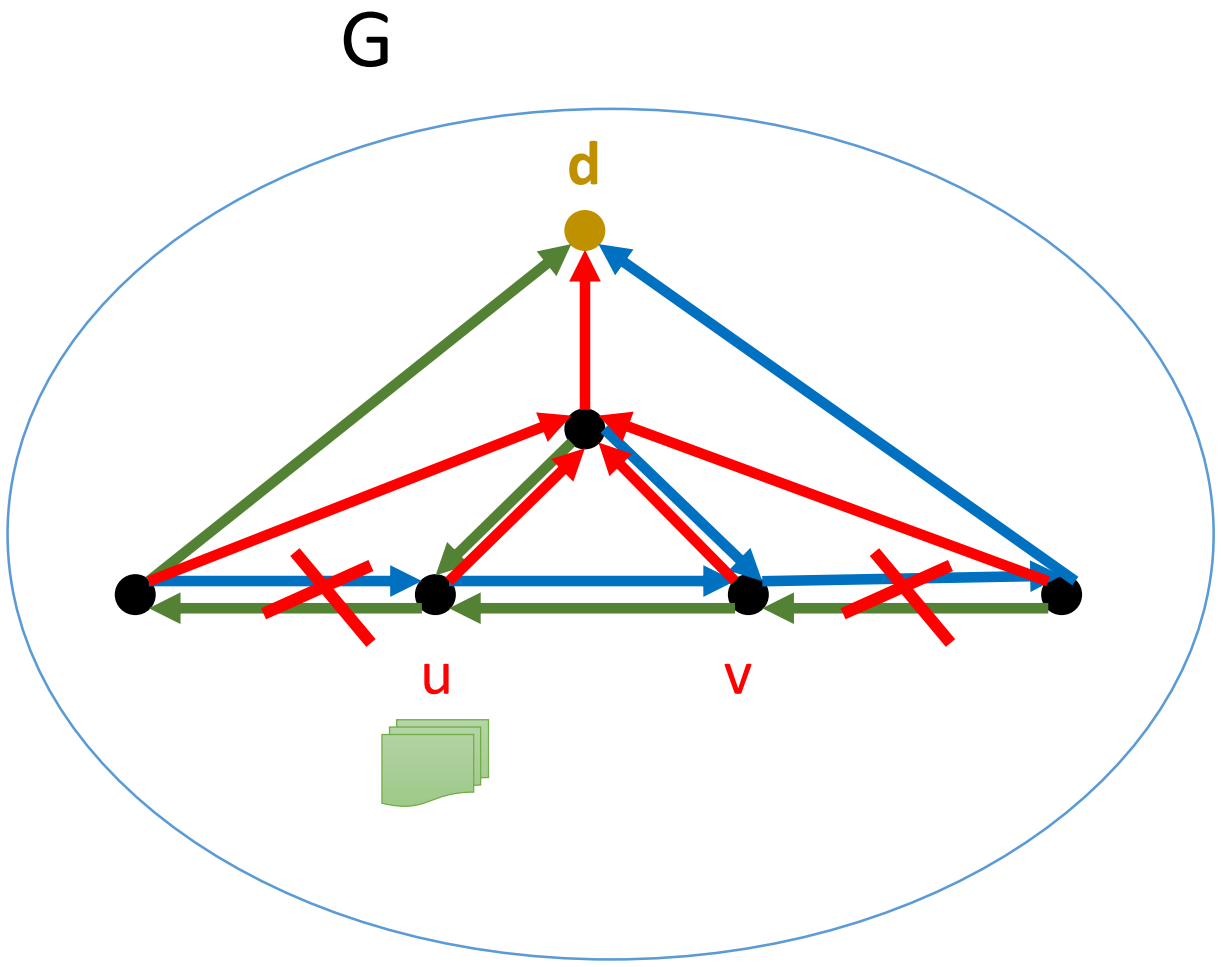

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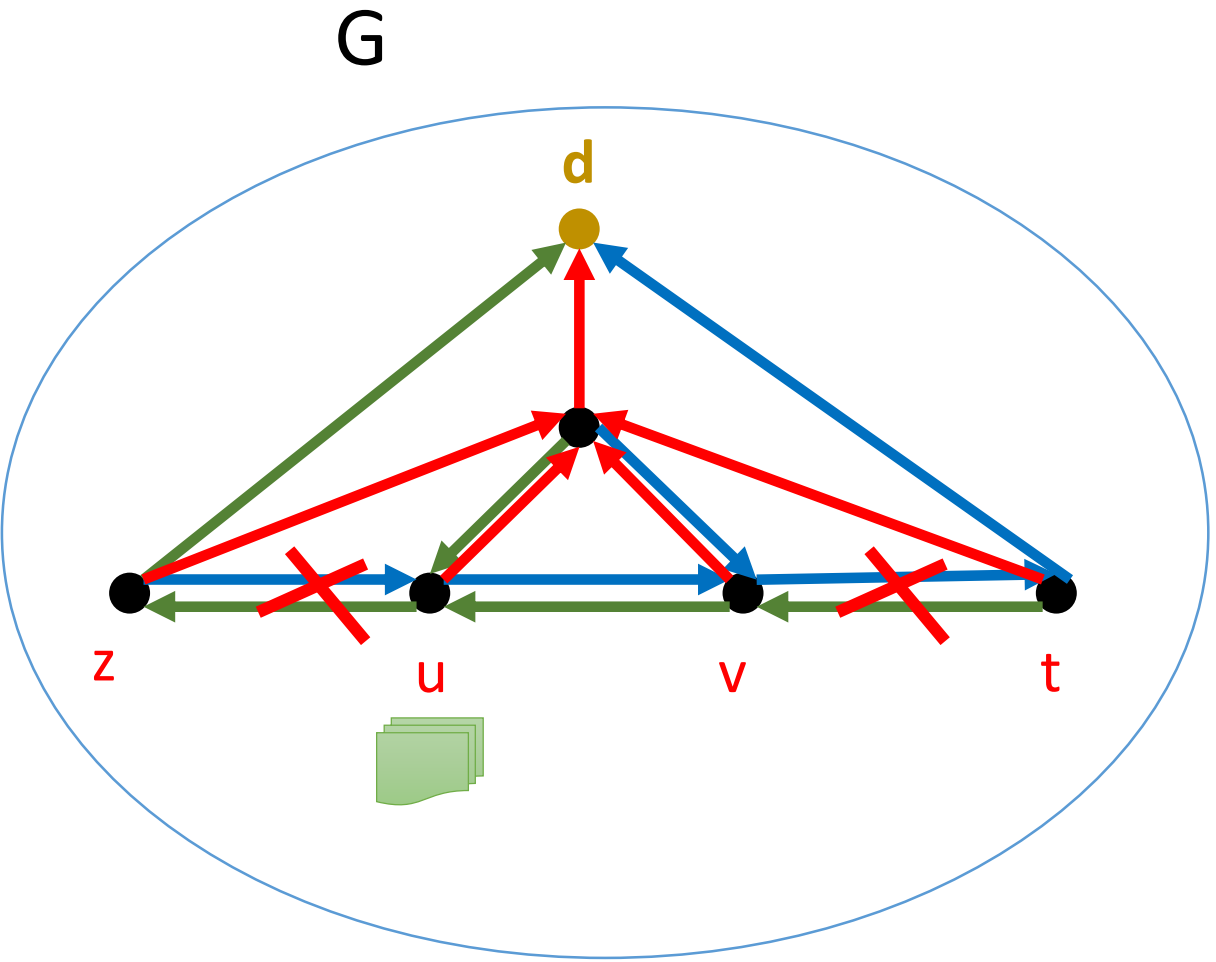
7. The packet is on blue at u.

LOOP!

Well-bouncing arcs and good arborescences

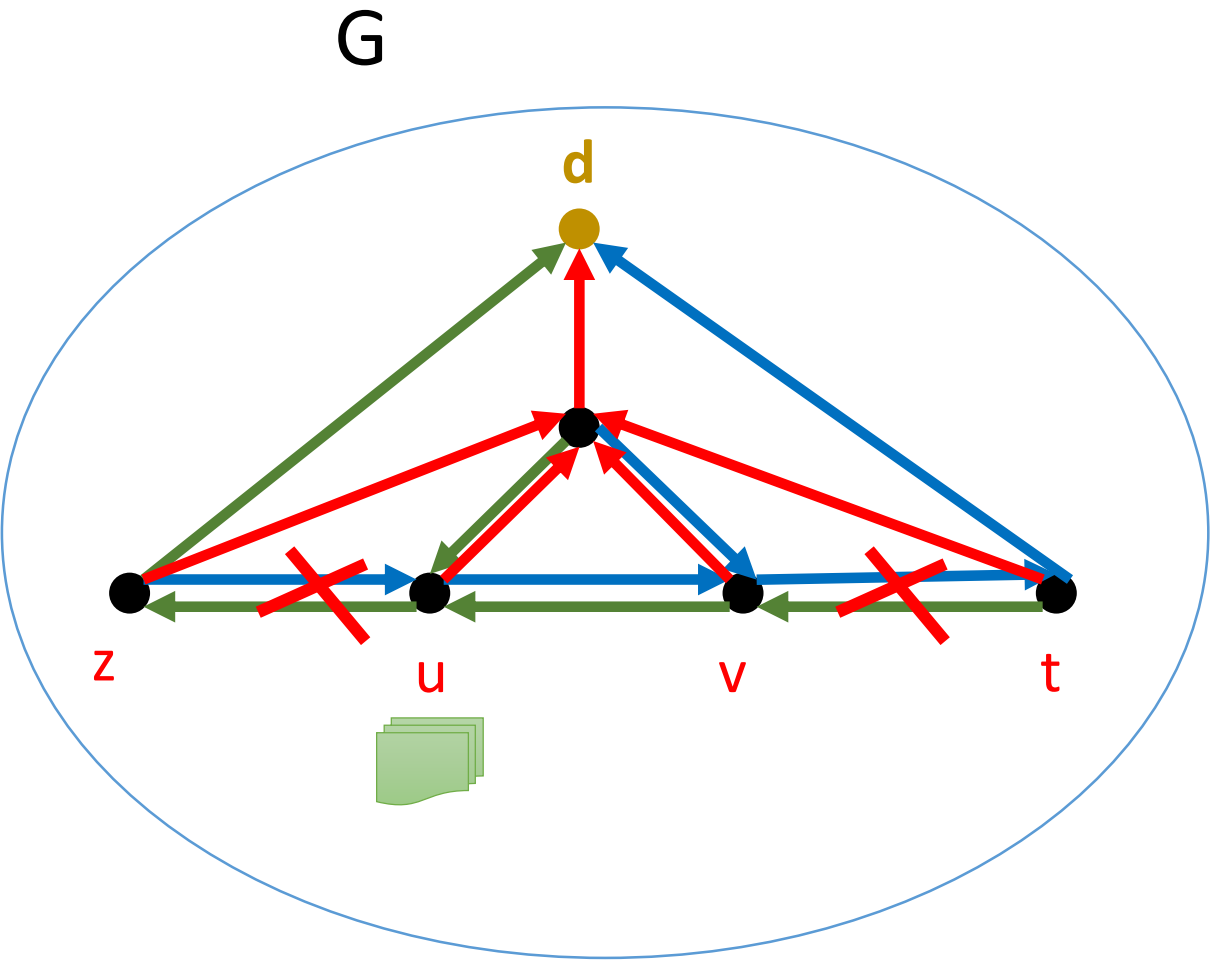


- An arc is *well-bouncing arcs* if bouncing on it takes a packet to **d** without any interruption. (<u>no more loops</u>)
- An arborescence is *good* if its every failed arc is well-bouncing.

(u, z) and (v, t) are <u>not</u> well-bouncing

(z, u) and (t, v) are well bouncing

Well-bouncing arcs and good arborescences



- An arc is *well-bouncing arcs* if bouncing on it takes a packet to **d** without any interruption. (no more loops)
- An arborescence is *good* if its every failed arc is well-bouncing.

We can show that such good arborescence always exists!

Goal now: identify good arborescence

When to bounce?

If we know the given arborescence is good, we can bounce.

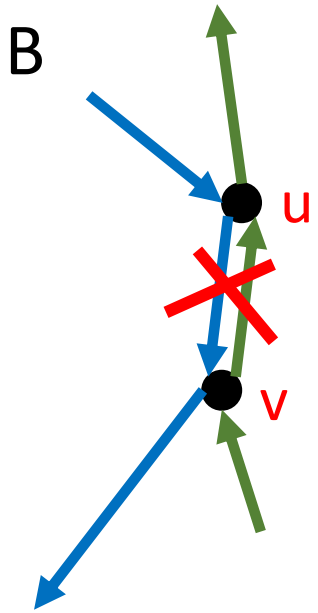
- Route along arborescence B
- If a packet hits a failed link:
  - If B is not good, route circularly
  - Otherwise, bounce

Too good to be true ... how do we know if an arborescence is good?

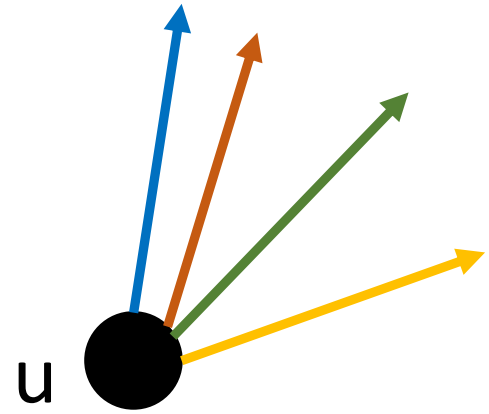
We don’t. We guess!

Goal: routing on a good arborescence

- Route along arborescence B
- If a packet hits a failed link make a guess



With prob *p*



With prob 1-*p*

(*p* is our likelihood that B is good)

Our main result

Theorem:

Let G be a k-connected graph, and A be a decomposition of G into k arc-disjoint arborescences rooted at **d**. Assume that there are at most f many failed edges. Then, A contains at least k-f good arborescenes.

(for details see our paper)

Efficiency of our algorithm

- *(k-1)*-resilient routing scheme using randomness.
- Number of failed links a packet hits is $\Theta\left(\frac{k}{k-f}\right)$ by choosing appropriate *p*. (more details in the paper)
- If $f < (1-\varepsilon)k$, for constant $\varepsilon < 1$, the packet hits only a constant number of failed links in expectation.

Open problems

- Can we devise deterministic (*k-1*)-resilient routing tables?
- Can we slightly alter the given network and improve the resiliency?
- Can we devise even a randomized (*k-1*)-vertex-resilient routing scheme, if the network is *k*-vertex-connected?