Nesting Depth of Operators in Graph Database Queries: Expressiveness vs. Evaluation Complexity

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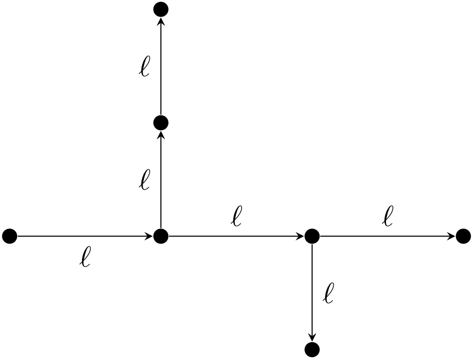
Regular Path Queries



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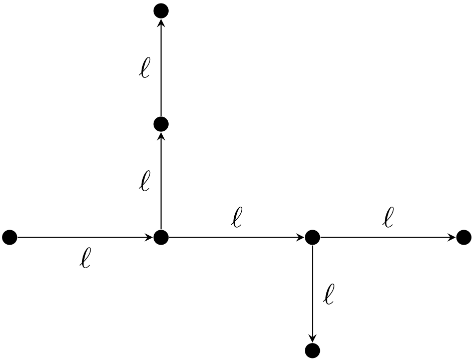

A regular path query: ℓ^*

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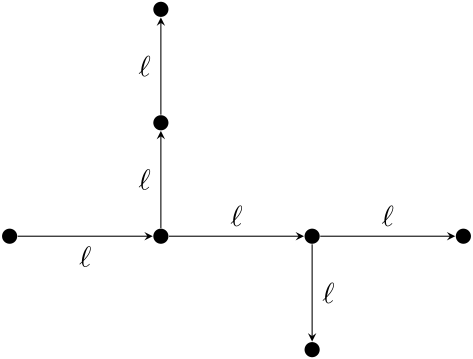

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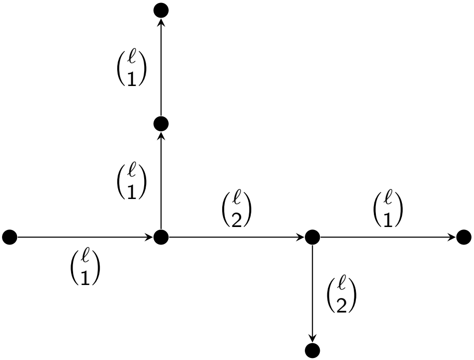
Regular Data Path Queries



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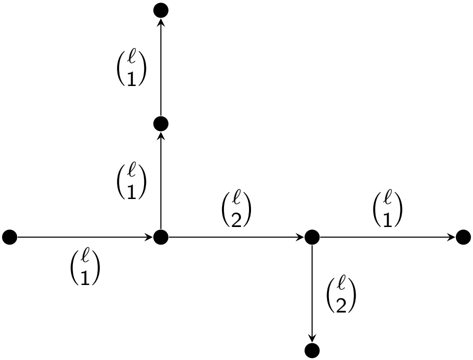

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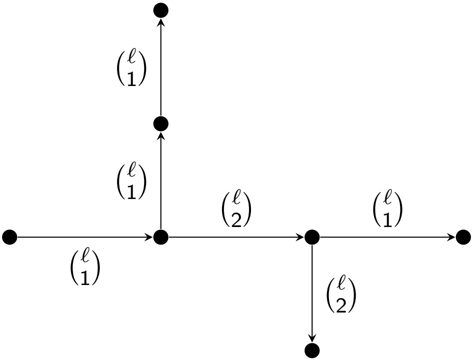

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Iterated Bindings

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Regular Expressions With Binding [Libkin, Tan, Vrgoč 2013]

$$r ::= \epsilon \mid a \mid a[c] \mid r + r \mid r \cdot r \mid r^* \mid a \downarrow_x (r)$$

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Results

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4. Controlled reductions from satisfiability of quantified Boolean formulas to query evaluation.

Automata View of Controlled REWBs

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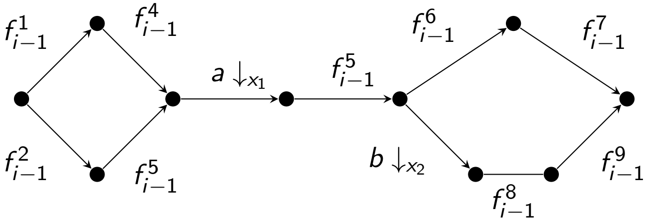
Automaton for e_i



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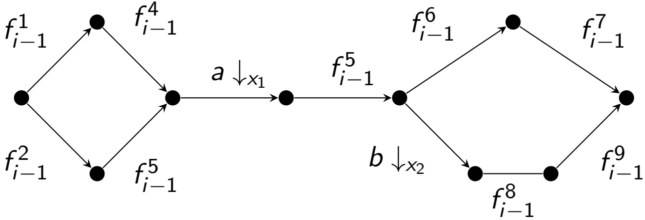

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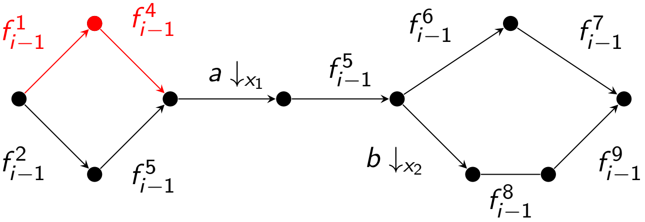

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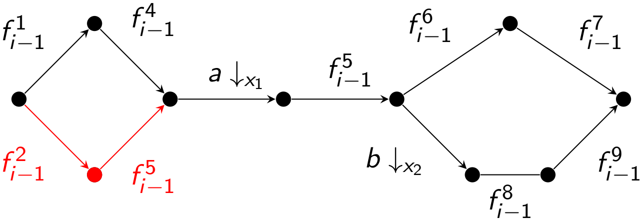

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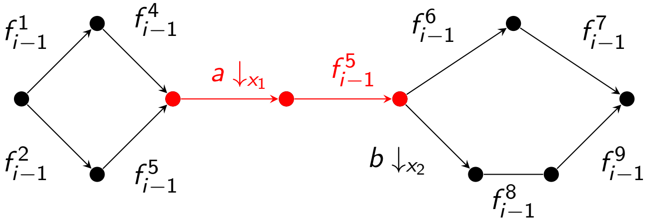

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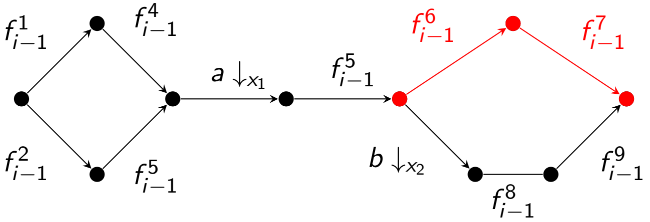

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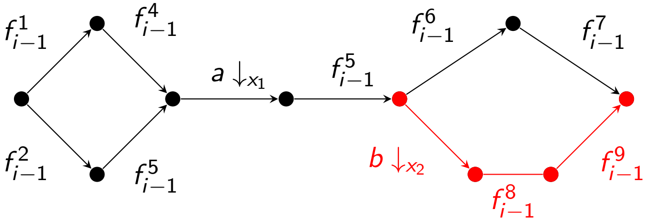

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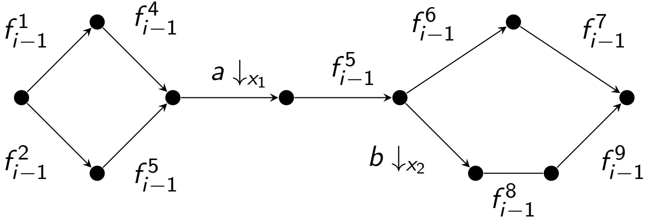

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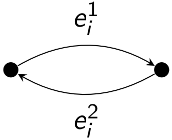
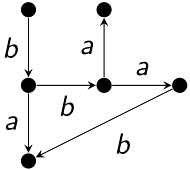


Evaluating F_i Queries Using Oracle for E_i



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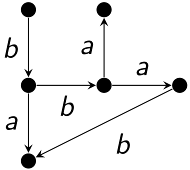

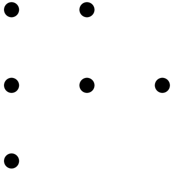

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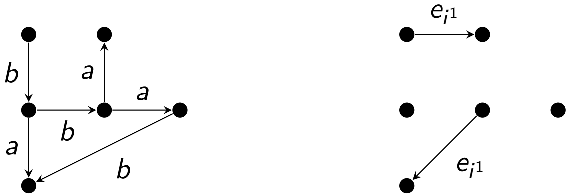

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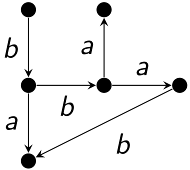

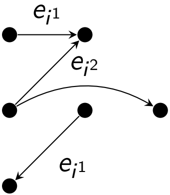

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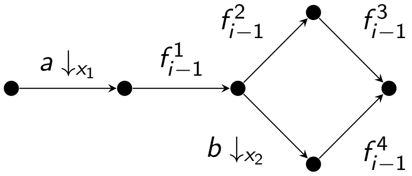
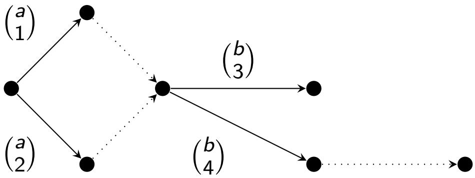
Evaluating E_i Queries Using Non-determinism





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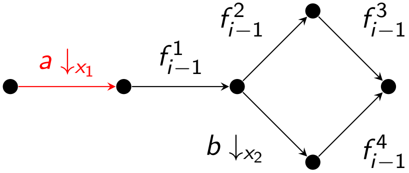

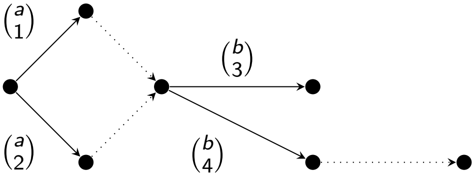

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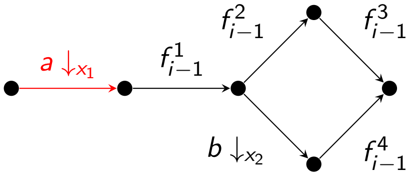

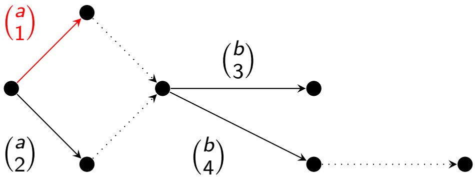

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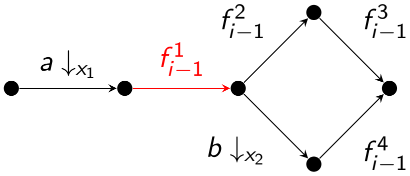

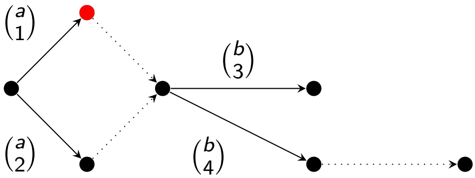

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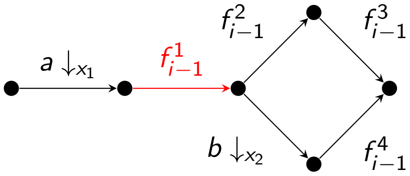

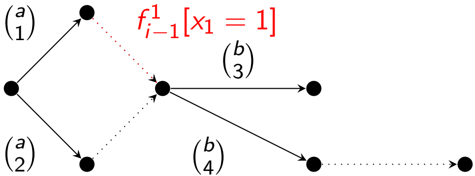

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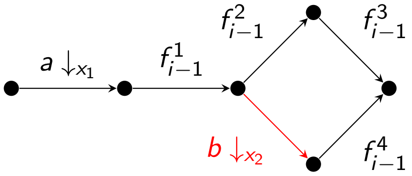

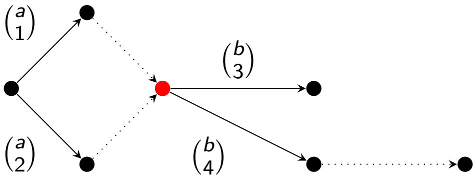

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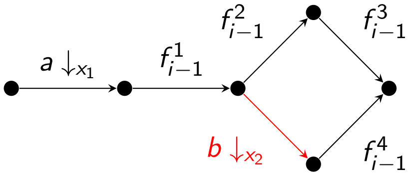

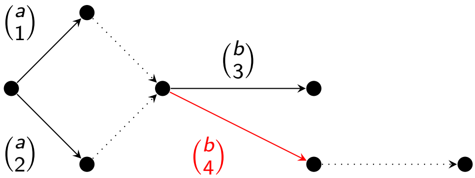

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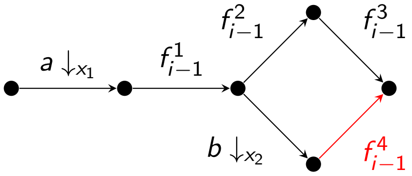

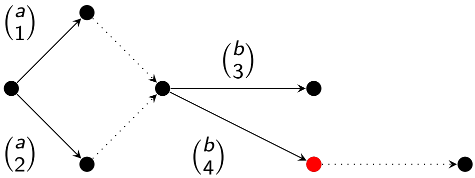

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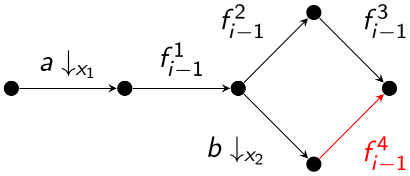

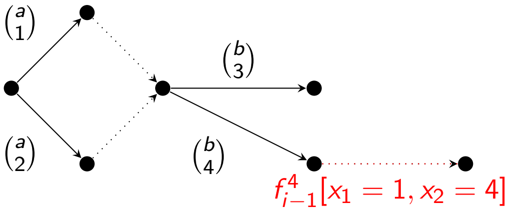

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Gap in Lower Bounds

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Thank You